Part 3
The Elements Project
Description of artistic output

Chapter 8. General description of the Elements Project

8.1 Introduction: aim of the Elements Project

Part 1 resulted in the construction of two formulas to determine the degrees of tonality and prime consonance of pitch class sets belonging to any set class. Part 2 provided an explanation of how these formulas represent or describe two of the endophysical laws of my aesthetic universe. Many of the ideas expressed in my music belong to the *idiosyncratic* part of my aesthetic universe, requiring a technique and idiom that is highly atonal and dissonant. This idiosyncratic part of my aesthetic universe is explored in Parts 1 and 2; these parts represent the 'space probe' necessary for the exploration of my aesthetic universe in order to make the expression of its ideas possible. At the same time the endophysical laws of my aesthetic universe developed in Parts 1 and 2 are themselves also aesthetic ideas—ideas the meaning of which can be expressed in composition. The CIG-technique with which I express (the meaning of) my aesthetic ideas is a *procedure* (procedural knowledge) as well as an *aesthetic idea* (conceptual knowledge) that is part of the aesthetic message I attempt to convey. The different ideas (conceptual and procedural) cannot be separated. Indeed, as was discussed in Section 7.1, in art, *what* is communicated is as important as *how* it is communicated. Therefore, the preceding theoretical parts (Part 1 and 2) are also part of the meaning of the ideas that I express in the compositions that constitute the artistic output of my research.

The core of the artistic output of the present research is formed by three orchestral pieces that express the *what* and the *how* of my aesthetic universe, and that illustrate three phases of my research: the initial situation at the start of the research, the intermediate situation with its provisional results, and the final situation implementing the adapted CIG-technique: general CIG-serialism.

The three orchestral pieces—*Danse de la terre* (2010), *Danse du feu* (2012) and *Danse de l'eau et de l'air* (2014)—are 'dances' representing or expressing[541] the four metaphorical Empedoclean elements (earth, water, air and fire) that stand for different aspects of my aesthetic universe and together encompass this aesthetic universe in its entirety. They are in this way synecdoches for the whole of my aesthetic universe in the way the four Empedoclean elements stand (or stood) for the complete physical universe. The four elements of my aesthetic universe are not the same as the physical elements that constitute the physical world we live in, but their endophysical counterparts that are (at most) *in* the image of the elements of the physical world. The aim of the artistic output of the present research was to express these aspects—the four elements of my aesthetic universe—in a non-verbal way. Hence the title of the project: the **Elements of an Aesthetic Universe Project**, or short: the **Elements Project**.

[541] Davies would say that the dances of the Elements cycle do not *represent* the elements of my aesthetic universe, they *express* the complete meaning of the concepts of those elements that belong to my aesthetic universe (see Stephen Davies' "Argument against Musical Representation", in Steven Davies, *Musical Meaning and Expression*, Cornell University Press, 1994, pp. 79-81).

Mahler is reported to have said that "a symphony should be like the world: all encompassing"[542]. On the other hand, Mahler also wrote in a letter from 1904 to Bruno Walter: "Music represents the whole human being—feeling, thinking, breathing, suffering."[543] A combination of both statements can be interpreted as meaning that a symphony should be the expression of the artist's complete aesthetic universe. In that sense my orchestral cycle on the Empedoclian elements may be called 'symphonic' in the Mahlerian sense; each dance expressing one 'element' (or two in the case of *Danse de l'eau et de l'air*) of my aesthetic universe.

The meaning of the aesthetic ideas expressed in the three orchestral dances of the Elements Project consists for the most part of non-verbal concepts (non-verbal ideas), and can therefore not be expressed in words. Still it is possible to give a rough impression of some of the ideas related to the pieces, albeit in a rudimentary and non-comprehensive manner: *Danse de la terre* expresses the idea of the (endo-)physicality of my aesthetic universe—matter (earth) as material existence—and by extension it expresses *existing* in general. *Danse du feu* is an expression of the idea of the endophysical processes governed by my aesthetic universe's laws of tonality and consonance. It is an expression of the idea of endophysical *becoming*. Together with *Danse de la terre*, this second dance expresses all that *exists*: matter and material processes. *Danse de l'eau et de l'air*, in turn, expresses the elusiveness entailed by matter and material processes: impermanence, time and temporality, transience, the fact that what exists could as well not have existed. It is about the contingency of *being*. One might also say that *Danse de la terre* expresses the aspect (or concept) of 'mass', *Danse de l'eau et de l'air* the aspect of 'time' and *Danse du feu* the aspect of 'energy' in my aesthetic universe.

Reference was made before to the poetic way in which Richard Feynman approaches physics.[544] The idea of blending scientific and aesthetic (poetic) thought reaches its zenith in the work of the French philosopher of science Gaston Bachelard (1884-1962). Many of Bachelard's works address the poetic dimension of the aspects of science, which he coincidentally also relates to the four Empedoclian elements. This is the case in such works as *La psychanalyse du feu* (1938), *L'eau et les rêves* (1942), *L'air et les songes* (1943), and *La terre et les rêveries de la volonté* (1948). In these works (and in many others) he freely links countless sources of literature with ideas that belong to the fields of sciences and psychoanalysis. One might even claim that Bachelard's "rêveries" represent an approach of free association in the Freudian psychoanalytical sense. The Elements Project—and as a matter of fact the entirety of the research described in the present text—does the opposite: starting from an aesthetic perspective, it integrates scientific elements; whereas Bachelard applies ideas from aesthetic universes (not just his own) to the physical world, the Elements Project borrows ideas belonging to science(s) in order to explore and express my own aesthetic universe.

Although the three orchestral dances are self-contained pieces that can be performed independently, together they form a trilogy, a cycle that can be performed as three movements of a single symphonic piece. In that case, for reasons of overall structure, the pieces should be performed in the following order: first *Danse de la terre*, then *Danse de l'eau et de l'air*, and finally *Danse du feu*. When performed as a cycle in this order, the size of the orchestra increases with every piece, as shown in Example 8.1.

title	**woodwinds**	**brass**	**percussion**	**strings**	**total**
Danse de la terre	3-3-3-3	6-3-2-1	5 perc - 1 pno	9-9-9-9-9	**75**
Danse de l'eau et de l'air	4-3-4-3	4-3-3-1	4 perc - 1 pno - 1 harp	12-12-10-8-6	**79**
Danse du feu	4-4-4-4	4-4-3-1	6 perc - 1 pno -1 harp	16-14-12-10-8	**96**

Example 8.1: Increasing size of the orchestra in the three orchestral dances of the Elements Project.

[542] "Die Symphonie muss sein wie die Welt. Sie muss alles umfassen". Gustav Mahler in a conversation with Jean Sibelius. Quoted in: Andrew Barnett, *Sibelius*, Yale University Press, 2007, p. 185.
[543] Herta Blaukopf (ed.), *Gustav Mahler Briefe: 1879-1911*. Rev. and enl. Ed. Publications of the International Gustav Mahler Society, Vienna/Hamburg, 1983. Quoted in: Constantin Floros, *Gustav Mahler, The Symphonies*, Breitkopf & Härtel, 1985, translated by Vernon & Jutta Wicker, Amadeus Press, 1993, p. 12.
[544] See Section 7.6.

Each of the three orchestral works in the Elements Project is complemented by one or two works for smaller ensembles or solo instrument (see Example 8.2 below), each exploring an additional aspect of the research. *Danse de la terre* is complemented by the ensemble piece *Le sourire infini des ondes* (2009), *Danse du feu* by the piano quartet *Un souffle de l'air que respirait le passé* (2011), and *Danse de l'eau et de l'air* by the two piano pieces *A l'image du monde…originel* (2012) and *A l'image du monde…double* (2013). Note that, in every phase, the composition of the complementary pieces precedes that of the orchestral piece they accompany; in a way they are preparative studies for the orchestral piece they complement. Each phase in the Project is discussed in greater detail next.

Phase	Orchestral works	Complementary pieces
Phase 1 (initial situation)	*Danse de la terre* (2010)	*Le sourire infini des ondes* (2009) (ensemble)
Phase 2 (intermediate results)	*Danse du feu* (2012)	*Un souffle de l'air que respirait le passé* (2011) (piano quartet)
Phase 3 (final outcome)	*Danse de l'eau et de l'air* (2014)	*A l'image du monde…originel* (2012) (piano) *A l'image du monde…double* (2013) (piano)

Example 8.2: Pairing of orchestral works and complementary pieces within the Elements Project.

8.2 Phase 1: *Danse de la terre* and *Le sourire infini des ondes*

The first orchestral work in the Elements-cycle, *Danse de la terre,* was commissioned by the Festival of Flanders and was first performed by the National Orchestra of Belgium on 10 September 2010 in the Henry Lebœuf Hall of the Paleis voor Schone Kunsten in Brussels during the KLARA festival and Festival of Flanders. The performance was live broadcast on KLARA (Flemish National Classical radio). The work illustrates the initial situation (phase 1) of my research. It is written with the original technique of CIG-serialism, as it was prior to the assessment and further development that was the subject of the theoretical part of my doctoral research.

Danse de la terre could be interpreted as a slow waltz, although the only element the piece has in common with a waltz is the fact that it is written in three-four time throughout (there are no changes in time signature). It is a dance of the earth at different levels: a dance of the element earth, the earth as raw material (and thus related to the primitive *Danse de la terre* in Stravinsky's *The Rite of Spring* (1913)), but it can also represent the motion of the Earth around the sun from two angles: if we look at the earth from a position outside the solar system, we witness a slow, elegant cyclic movement around the sun, like a very slow dance of one revolution per year, but when we bear in mind the fact that the earth weighs a sloppy 6 x 10^{21} tons and moves at more than 100,000 kilometers per hour, every romantic connotation quickly disappears. *Danse de la terre* evokes both images simultaneously ('blending' of interpretations, in other words).

There might be a connection between *Danse de la terre* and Gustav Mahler's *Das Lied von der Erde* (1908-1909). Theodor Adorno claims that, as a result of the integration of Chinese poetry into a Western Romantic symphony, *Das Lied* represents a pseudomorph[545] that Mahler used as a 'false form' or mask for his "Jewishness"[546] to hide his lack of worldly 'roots', and obtain an 'effect of alienation'

[545] Theodor W. Adorno, *Mahler: Eine Musikalische Physiognomik*, 1960, in *Die Musikalischen Monographien*, Suhrkamp, 1997, p. 291. The concept of pseudomorph is used in mineralogy to indicate "a mineral having the characteristic outward form of another species." (Merriam-Webster's Collegiate Dictionary).

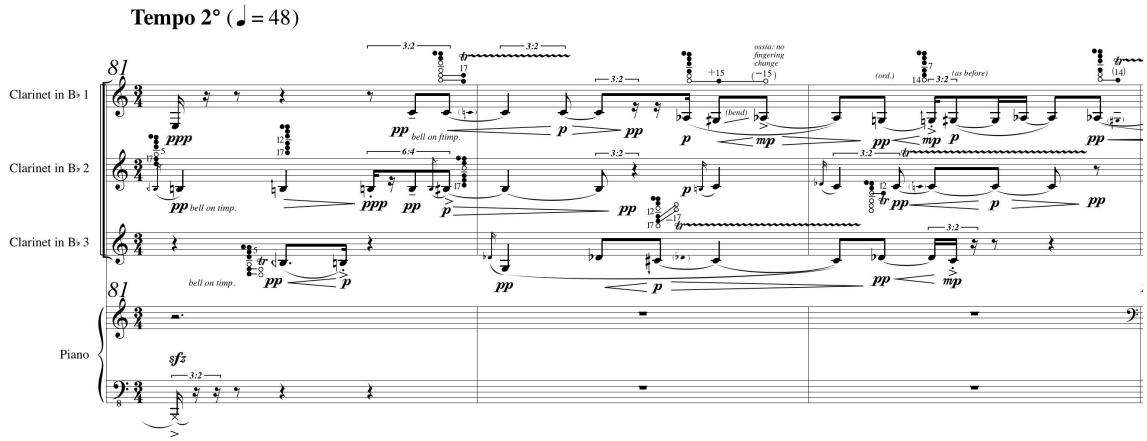
[546] "Pseudomorphose ist dieser Osten auch als Deckbild von Mahlers jüdischem Element." Theodor W. Adorno, *Mahler: Eine Musikalische Physiognomik*, 1960, in *Die Musikalischen Monographien*, Suhrkamp, 1997, p. 291.

(Verfremdungseffekte)[547]. *Danse de la terre*, on the other hand, may be interpreted as an expression of attachment to the physical world, to 'the earth'. "According to Claude Lévi-Strauss, through music we become conscious of the physiological roots of our being."[548] Are not both works then about the transcendent sublimity of earthly beauty? Does the claim that "there must be more between heaven and earth" not grossly underestimate the inherent transcendence (a contradiction?) of the physical world? As if the physical world as it is would be 'incomplete'. As if mysticism would not be possible without metaphysics. *Danse de la terre* is a manifesto for the revaluation of physical reality and our total commitment to that endophysical reality. That way, the earth is a synecdoche for everything that exists. The earth with its beauty and its horror, that's what we are.

The numbers 3, 6 and 9 play an important part in the structure and orchestration of *Danse de la terre*. The instrumentation consists of groups of three, six or nine instruments: three flutes, oboes, clarinets, bassoons and trumpets, two trombones and one tuba (forming a group of three low brass instruments), six horns, five percussions and one piano (six together). Within the percussion instruments there are six gongs, and three of several instruments (three bass drums, three brake drums, …), and finally nine instruments in each group of the string quintet. There is clearly a shift in balance towards the low instruments within the string group with its nine double basses.

The piece has three sections of 80 bars each. In each section one instrumental group stands out: the first section starts with the nine solo-violas in a nonet, in the second section the three clarinets play the prominent part, and the third section has the six horns as its protagonists.

The instruments within the woodwind section are deliberately limited to their standard instruments: flutes but no piccolo's, alto flutes, or bass flutes; oboes, but no English horns or oboe d'amore, clarinets in B flat, but no other clarinets or bass clarinets, bassoons, but no double bassoons. *Danse de la terre* explores the possibilities of the woodwinds whilst limiting the instruments employed. To enhance the tone colour possibilities of the clarinets in their solo section (the beginning of section 2), they are requested to play with their bell on a kettledrum (*bell on timp*) and to modulate the tone by pressing and depressing the kettledrum's pedal (see Example 8.3).

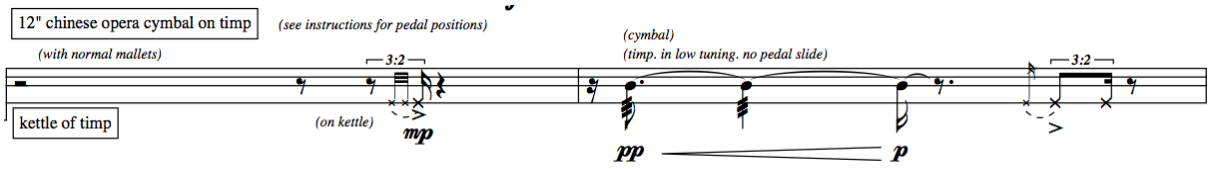


Example 8.3: Beginning of section 2 (bars 81 to 83) of *Danse de la terre* with three solo clarinets playing with their bells on a kettle drum.

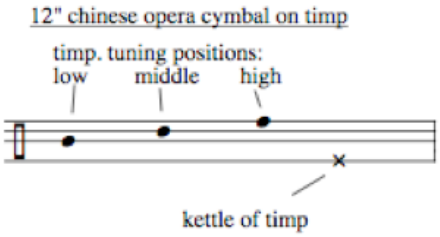
Le sourire infini des ondes, the complementary piece to *Danse de la terre*, is written for an ensemble or nine performers and was commissioned in 2009 by the Spectra Ensemble. It was first performed by this ensemble in the Academiezaal (Academy Hall) in Sint-Truiden (Belgium) on 12 May 2010 and in the concert hall of De Bijloke in Ghent (Belgium) the next day.

[547] Theodor W. Adorno, *Mahler,* p. 291.
[548] Eero Tarasti, *La Musique et les Signes*, L'Harmattan, 2006, p. 131 [my translation].

The piece is scored for bass flute, bass clarinet in B flat, horn in F, percussion (1 player), piano, and string quartet. The choice for low woodwinds reflects the shift towards the low instruments in the string section of *Danse de la terre*. The percussion instruments are restricted to metal instruments (different kinds of cymbals, tam-tams and gongs). This corresponds with the restriction of the woodwind section in *Danse de la terre*. The parallel between the two pieces is drawn further with the addition of a kettledrum: the kettledrum in *Le sourire infini des ondes* is never played on the skin (compare with the kettledrums used as a resonator for the clarinets in *Danse de la terre)*; it is only struck on the kettle or used to modulate the sound of the 12" Chinese opera cymbal placed on the skin of the kettledrum (see Example 8.4 a & b). In addition, the wind instrument players and pianist play a triangle in the middle section of the piece (bars 69-83).



Example 8.4 a: Percussion part of *Le sourire infini des ondes* (bar 101-102).



Example 8.4 b: Notation of pedal positions for kettledrum in *Le sourire infini des ondes*.

As will be discussed in the detailed analysis of Chapter 9, *Le sourire infini des ondes* is largely based on elements from Luigi Nono's piece *...sofferte onde serene…* for piano and tape (from 1976). In Nono's piece, the tape is entirely constructed with recorded (and often manipulated) sounds played by the Italian pianist (and friend of Nono's) Maurizio Pollini on the piano. This idea was transposed to *Le sourire infini des ondes*: Although *Le sourire infini des ondes* is a piece for ensemble without solo instrument, the piano plays a prominent structural role in the piece. The sound was conceived with the piano as the generating starting point. All the tone colours are deduced from the piano part.

8.3 Phase 2: *Danse du feu* and *Un souffle de l'air que respirait le passé*

The second orchestral work at the core of the artistic output of Elements Project, *Danse du feu*, is based on intermediate results of my research (phase 2). After developing formulas for the quantification of tonality and dissonance, I investigated the possibility to construct amotivic series based on CIG-4's (instead of CIG-3's, as in the original CIG-serialism) in order to increase their degree of atonality and dissonance. This proved practically unfeasible (as was discussed in Section 5.4.1), but serendipity lead to the addition of two new 'CIG-3's' to the series (3-note CIG-2's), resulting in series with 56 CIG's instead of 54 (see Section 5.4.5.2). *Danse du feu* is based on such a 56-CIG series.

The piano quartet *Un souffle de l'air que respirait le passé* (finished in 2011), not only uses the same CIG series as *Danse du feu* (the work for which the quartet is a complement), but also the same RHS. The two works sound quite differently however, and have a very distinct expressive content, a different narrative. This way I want to refute the claim that the strict structural basis of my compositions reduces my work to solving a puzzle with no artistic dimension, and to show that even in

strict serial music, artistic creativity is decisive for the final result, and that there is a distinction between technique, idiom and style.[549]

The title of the piano quartet is the French translation of a fragment of a sentence from the Prologue of Luigi Nono's *"Tragedia dell'ascolto" Prometeo*: "Ascolta, non vibra qui ancora un soffio dell'aria che respirava il passato?"[550]. The fact that Prometheus is the titan who stole fire from heaven, links the quartet to the orchestral work *Danse du feu* (dance of fire).

Un souffle de l'air que respirait le passé was first performed by the piano quartet Tetra Lyre on 25 January 2012 in the Concertgebouw Brugge (Belgium) in the context of its *Prometheus Geketend* (Prometheus bound) project.

8.4 Phase 3: *Danse de l'eau et de l'air*, *A l'image du monde…originel* and *double*

The third orchestral work at the heart of the Elements Project is *Danse de l'eau et de l'air*. The work is complemented by the piano pieces *A l'image du monde ... originel* and *A l'image du monde ... double*. All three works use the modified CIG technique that is the outcome of my research (phase 3). The two piano pieces are examples of CIG-3/4-serialism, but also coincidentally of general CIG-serialism. Indeed, although the series were not conceived as such (see below), they are general CIG series. The series of *Danse de l'eau et de l'air* is a general 56-CIG series.

A l'image du monde… originel and *A l'image du monde… double* are also part of a larger future cycle of five pieces that will further consist of *A l'image du monde… multiple* for guitar solo, *A l'image du monde… commentaire,* for guitar and/or piano and ensemble, and the electro-acoustic 'composition' *Improvisation fixe sur une image*, which was 'composed'[551] in 2012 in the context of Kathleen Coessens' *A Day in my life*-project at the ORCiM in Ghent. *Improvisation fixe sur une image* is based on a bass flute improvisation on the first series constructed with the adapted technique of 3/4-CIG-serialism, in order to explore the possibilities of such series and to assess the difference with my previous series. It was meant to experience the intuitive embodied effect the new series might have on me during the process of composition. The *A l'image du monde* project wants to express the idea that works of art are "in the image of the world"[552]. It explores the connections between the physical universe and my personal aesthetic universe, bearing the motto: "All aesthetic ideas are fictitious; any resemblance with the physical world is unavoidable however". In this context, the following statement by Benedetto Croce is certainly worth quoting:

> Every true artistic representation[553] is in itself the universe, the universe in the individual form, and the individual form as the universe; in the accents of the poet, and in every creation of his mind, the entire destiny of mankind can be found, with all its hopes, all its illusions, its pains, its joys, its greatness, and its human misery; the entire drama of reality that arises, continuously grows, in suffering and in pleasure.[554]

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[549] For a discussion on the difference between technique, idiom and style, see: Bart Vanhecke, *Chromatic Interval Group Serialism, The Development of an Atonal, Dissonant, and Amotivic Composition Technique*, unpublished master thesis, 2014, LUCA, Leuven, pp. 45-9.

[550] "Listen, doesn't a breath of air that the past respires vibrate here?", Massimo Cacciari, libretto for Luigi Nono, *Prologo* of *Prometeo; Tragedia dell'ascolto,* 1981-1985 [my translation].

[551] 'Composed' is here written between quote signs because *Improvisation fixe sur une image* is strictly speaking not a composition but a recorded and manipulated improvisation. Therefore I do not include the piece in my work list.

[552] See Section 7.5.

[553] I would add: "or expression".

[554] Benedetto Croce, *Il Carrattere di Totalità dell'espressione artistica*, in *La Critica. Rivista di Letteratura, Storia e Filosofia diretta da B. Croce*, Vol. 16, Gius. Laterza & Figli, Editori, 1918, p. 131 [my translation] online: http://ojs.uniroma1.it/index.php/lacritica/view/7454/7436 [last accessed: 08 December 2012].

There is a clear reference to works of Pierre Boulez such as *Mémoriale (..Explosante-Fixe… Originel)* or *Figures-Doubles-Prismes* in the titles of the pieces belonging to the *A l'image du monde* cycle. It is not only the idea of interconnection between works that I wanted to adapt, but the reference was also made because of the correspondence I feel—it is indeed only an intuitive feeling— between the aesthetic universe of Pierre Boulez (and of Schoenberg and Nono for that matter) and my own.

8.5 Additional aspects

In addition to the aspect of the Empedoclian Elements and the three phases in my research, there are many other aspects and cross references that interconnect the pieces of the Elements Project.

First, there are the cross references in the titles connecting orchestral and complementary pieces of different phases: the aspect of 'earth' (or 'world') in the titles of *Danse de la* ***terre*** and *A l'image du* ***monde***, the aspect of 'water' (and waves) in *Danse de* ***l'eau*** *et de l'air* and *Le sourire infini des* ***ondes***, and the aspect of 'air' in *Danse de l'eau et de* ***l'air*** and *Un souffle de* ***l'air*** *que respirait le passé*. Another cross reference is provided by the fact that *Le sourire infini des ondes* and *Un souffle de l'air que respirait le passé* share a common reference to Luigi Nono's *...sofferte onde serene...* and *Prometeo.*

Other links between the pieces in the Elements Project are created by the role of the numbers 3 and 4, referring to the structure of 3/4-CIG-serialism as well as general CIG-serialism, which is based on the use of CIG-3's and 4's. This is not only reflected in the instrumentation of the orchestral pieces (as was mentioned before), but also in the fact that there are three orchestral works on four elements; four complementary pieces with three instrument combinations. The fact that one of the dances expresses two elements (*Danse de l'eau et de l'air*) is mirrored by the fact that the two piano pieces complement one orchestral dance.

I want to end this chapter on a personal note. The three orchestral dances are dedicated to my two daughters (*Danse de la terre* to my eldest, and *Danse du feu* to my youngest) and my wife (*Danse de l'eau et de l'air*). There is an intuitive associative link between the meaning of the concepts of earth and fire and the characters of each of my two daughters. Likewise, certain aspects of my wife's personality may be associated with the element water; I myself seem to fit the element of air better. These associations show how my personal universe blends with my aesthetic universe.

Chapter 9. Analyses of the Elements Project pieces

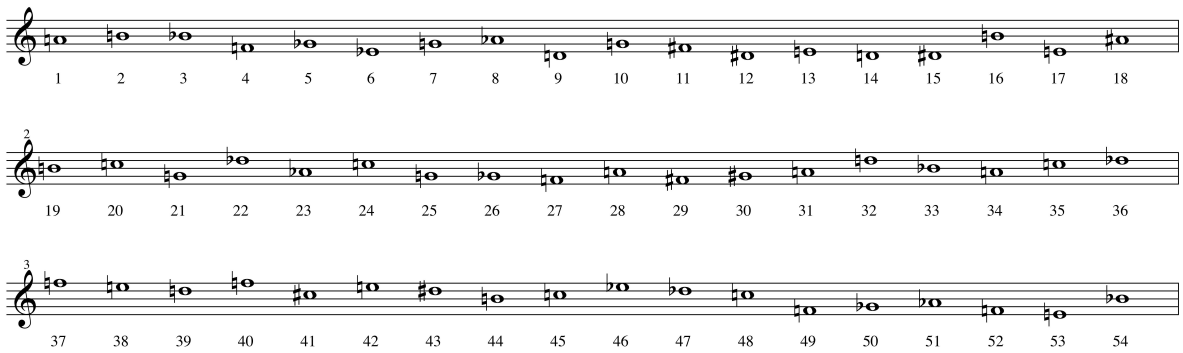
9.1 Preliminary remarks

The analyses in the present chapter do not provide a comprehensive explanation of all aspects of the seven pieces belonging to the Elements Project. Instead, they provide a description of specific aspects that are crucial in all pieces. The principles behind these aspects (such as the way series are constructed) are recurrent in all pieces. Therefore the sum of all aspects addressed in the present part gives a general view of all of the central issues I deal with in my artistic practice as a composer during the construction of series and RHS. The analyses of the first two pieces (*Danse de la terre* and *Le sourire infini des ondes*) focus on the way the series are constructed, and on levels of RHS and score (surface structure). The analysis of *Danse de l'eau et de l' air* focuses on the construction of a general CIG-series. The other analyses (*Un souffle de l'air que respirait le passé* and *A l'image du monde…originel* and *double*) focus on relations between the pieces belonging to the same research phase; they emphasize the relative importance of series and RHS on the final surface structure.

9.2 *Danse de la terre*

9.2.1 General structure and series

The series of *Danse de la terre* (see Example 9.1) is the same as the one I used for all my previous compositions from *Les racines du monde* for piano solo (1997) on, but in retrograde, transposed three semitones down and starting on the thirteenth series note. It is the last piece (to date) composed on the series of *Les racines du monde*.[555]

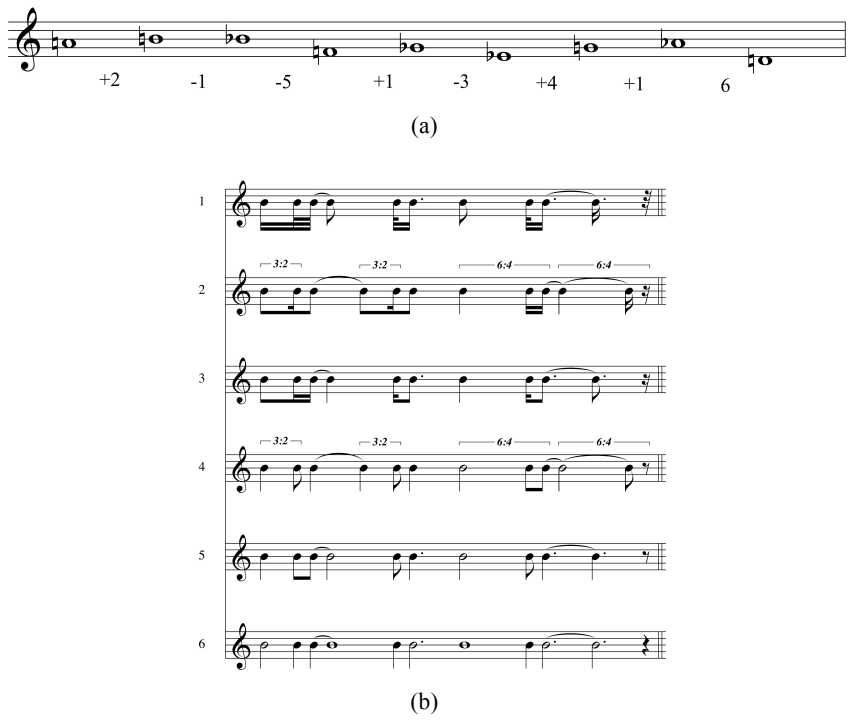


Example 9.1: Series of *Danse de la terre*.

[555] Originally, I called the series on which all my pieces between *Les racines du monde* and *Danse de la terre* are based 'the series of silence' because it was meant to serve as the basis for a never written chamber opera *Silence* on the play by Harold Pinter with the same name.

9.2.2 Rhythmic structure

The rhythm of *Danse de la terre* is based on seventeen rhythmic cells and their six augmentations (following the rhythmic chart in Example 1.14). The rhythmic cells were determined on the basis of the 'longest' ordered subsets of the series without repeated notes. Starting from the first note in the series, the first repetition of a pitch class occurs at note 10 (G), which is the same as note 7, as can be seen in Example 9.1. If the second note of the series were taken as a staring point, the resulting subset would contain only notes 2 through 10, which is one note less than in the subset starting from note 1. The latter is therefore the 'longest' subset and is kept as a basis for a first rhythmic cell (cell 1). The unordered interval class content between the successive notes in the longest subset (shown in Example 9.2 a) is used to determine the note lengths in each augmentation of cell 1 (shown in Example 9.2 b).



Example 9.2: (a) Ordered interval class content between the first nine notes of the series of *Danse de la terre*, and (b) rhythmic cell 1 based on the unordered interval class content with its six augmentations.

Example 9.3 lists the 17 'longest' subsets in the series and the unordered intervals contained in each of them. The list shows that subset 1 is the longest of all 'longest' subsets.

Rhythmic cells and their augmentations are attached to the series notes in the following way. The first rhythmic cell uses cell 1 in augmentation 1. To determine next rhythmic cell for the next note, the number of the previous cell is augmented with the sum of the unordered interval classes preceding and following the note (modulo 17). The unordered interval classes surrounding note 2 are 2 and 1 (see Example 9.2 a), therefore its rhythmic cell is cell 4 (cell 1 + 2 + 1 (mod 17)). The augmentation of the rhythmic cells of the next series notes is determined by the increasing the previous augmentation with the ordered interval class preceding the note (modulo 6). The ordered interval class between note 1 and 2 is +2, therefore the augmentation for the rhythmic cell of note 2 is 3 (1 + 2 = 3 (mod 6)). This system is implemented over the whole series. When the end of the series is reached it starts again from the beginning. Deliberate mutations (deviations from the rules,[556] when an extra '1' is added to the sum in cell determination) occur in this structure at more or less regular intervals.

[556] See Section 1.2.

subset number	first note	last note	interval content
1	1	9	2 1 5 1 3 4 1 6
2	8	13	6 5 1 3 1
3	10	14	1 3 1 2
4	13	16	2 1 4
5	14	18	1 4 5 6
6	17	23	6 1 1 5 6 5
7	21	24	6 5 4
8	22	28	5 4 5 1 1 4
9	27	30	4 3 2
10	29	33	2 1 5 4
11	32	38	4 1 3 1 4 1
12	33	39	1 3 1 4 1 2
13	38	41	2 3 4
14	39	45	3 4 3 1 4 1
15	44	47	1 3 2
16	46	51	2 1 5 1 2
17	50	2	2 3 1 6 1 2

Example 9.3: Range (from first note to last note) of the 17 'longest' subsets without note repetition in the series of *Danse de la terre* and their unordered interval content.

The distribution of the rhythmic cells is done freely in *Danse de la terre* according to the necessities of each moment. The RHS was therefore constructed at the moment of composition (*Auskomponierung*) of the surface structure. Attention was paid to harmonic and rhythmic balance in this process, as well as to overall texture and density.

9.3 *Le sourire infini des ondes*

9.3.1 Title

The title of *Le sourire infini des ondes* is taken from Luigi Nono's explanatory notes accompanying his piece *...sofferte onde serene...* for piano and tape from 1976. In these notes, Nono writes about the *"endless smile of the waves"*[557]. Nono himself places the expression between quotation marks because it is a quote from *Prometheus bound* by Aeschylus :

> O divine air Breezes on swift bird-wings,
> Ye river fountains, and *of ocean-waves*
> *The multitudinous laughter* Mother Earth!
> And thou all-seeing circle of the sun,
> Behold what I, a God, from Gods endure![558]

9.3.2 Elements from *...sofferte onde serene...*

The structure of *Le sourire infini des ondes* is based on the following elements from *...sofferte onde serene...*:

[557] Luigi Nono, *Ecrits.* Laurent Feneyrou (ed. & trans.), Paris: Christian Bourgois Editeur, 1993, p. 320.
[558] Aeschylus, *Prometheus* bound, http://classics.mit.edu/Aeschylus/prometheus.html [my italics][last accessed: 10 February 2012].

a) Form and duration

...sofferte onde serene... consists of nine sections, separated by eight "tape reference points" (*riferimento nastro*) indicated in the score. Nono's indications of the precise moment of occurrence of each of tape reference point in the score are listed in Example 9.4.

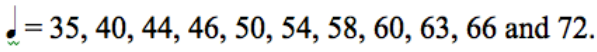
reference point	time
1	54"
2	1'56"
3	2'57"
4	5'11"
5	6'49"
6	9'16"
7	11'49"
8	13'14"

Example 9.4: Tape reference points in the score of *...sofferte onde serene...*

The total duration of the piece is 13'58".[559]

b) Tempo structure

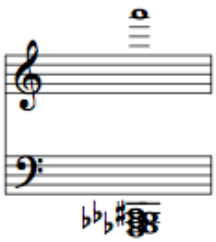
The tempo of *...sofferte onde serene...* fluctuates constantly between the following speeds:



In addition, numerous tempo changes occur in *...sofferte onde serene...* in the form of rallentandi and accelerandi, especially in the first two sections of the piece.

c) The first chord

...sofferte onde serene... starts with the chord represented in Example 9.5.



Example 9.5: First chord of *...sofferte onde serene...*

[559] 13'58" is the exact duration of the recording of the piece made by Maurizio Pollini in 1979. Pollini was the pianist who worked in close collaboration with Nono during the composition of the piece and the recording of the tape. Therefore, this recording may be considered a reference.

This chord contains the pitch classes of a pitch class set with Forte number [6-14] with interval vector <323430>. Note the absence of ic 6 in the interval vector.

9.3.3 Form of *Le sourire infini des ondes*

Le sourire infini des ondes consists of nine sections that correspond to the nine sections of *...sofferte onde serene...*. They contain the same number of crotchet beats as each corresponding section in Nono's piece, as is shown in the list below (Example 9.6).

section	number of crotchet beats
1	39
2	39
3	20
4	96
5	96
6	112
7	144
8	56
9	16

Example 9.6: Number of crotchet beats in each section of *...sofferte onde serene... and Le sourire infini des ondes*.

Although the number of beats in each section of *Le sourire infini des ondes* is exactly the same as in the corresponding section of *...sofferte onde serene...*, the grouping of the beats in bars is different. This grouping is adapted in order to make tempo indications and changes coincide with the beginnings of bars as much as possible. The time changes in *Le sourire infini des ondes* are therefore not related to metre (since the piece is essentially a-metric) but to the tempo structure of the piece.

9.3.4 Tempo structure and duration

a) Tempo structure

The tempo structure of *...sofferte onde serene...* is adopted unchanged in *Le sourire infini des ondes*, with exactly the same tempo changes and pauses (fermata). The position of the fermata in the piece may differ slightly from that of *...sofferte onde serene...*.

b) Duration in motion

Starting from the tempo structure within each section of *...sofferte onde serene...,* but disregarding the fermata, and assuming that tempo changes happen in a linear way (in order to be able to use the average of the highest and lowest tempo in each tempo change), the 'duration in motion' of each section can be calculated. The duration in motion of a section is its duration without the fermata.

For instance, Section 6 'moves' during 52 beats at the tempo72 for a crotchet first. This is followed by a linear rallentando from 72 to 50 during 8 beats. The tempo remains 50 during 32 beats and then continues at tempo 32 for 20 beats as shown in the list in Example 9.7.

tempo	number of crotchet beats
♩ = 72	52
♩ = 72 → ♩ = 50	8
♩ = 50	32
♩ = 35	20

Example 9.7: Number of crotchet beats in Section 6 in different tempos.

The 'duration in motion' of Section 6 can be calculated as follows (taking 61 to the crotchet as the average tempo during the rallentando):

(60" x 52 / 72) + (60" x 8 / 61) + (60" x 32 / 50) + (60" x 20 / 35) = 123,89"

The duration in motion of section 6 is therefore (approximately) 125 seconds. The duration of the other sections is calculated in the same way.

c) Pauses

The difference between duration in motion and (real) duration of each section between the points of reference is the time that has to be allocated for the pauses. The table below (Example 9.8) shows a complete overview of the time distribution of *...sofferte onde serene...*:

section	total duration	'duration in motion'	total duration of pauses	number of crotchet beats
1	54"	51"	3"	39
2	62"	43"	19"	39
3	61"	34"	27"	20
4	134"	95"	39"	96
5	98"	50"	48"	96
6	147"	125"	22"	112
7	153"	154"	-1" (!)	144
8	85"	84"	1"	56
9	44"	24"	20"	16
total	838"	658"	180"	618

Example 9.8: Time distribution of *...sofferte onde serene....*

Note that Section 7 has a duration in motion that lasts 1 second longer than the total duration and that, as a result, there is no time left to add a fermata. This can only be done when the tempi of this section are made a little higher than prescribed, but this can be no objection, since all tempi in *...sofferte onde serene...* are marked "circa".

I opted not to determine the length of the fermata for each section separately (a.o. because this creates the problem in Section 7 mentioned above), but to distribute the entire duration of the fermata (180 seconds) over all the fermata of the piece on the basis of the following criterion.

The whole piece contains 51 pauses of two kinds: short and long, indicated in the score as shown in Example 9.9.

Short pause	:	
Long pause	:	

Example 9.9: Notation of short and long pauses.

The pauses are distributed over the sections as indicated in the table of Example 9.10 below.

Section	Number of	Number of
1	4	1
2	1	1
3	2	2
4	19	9
5	5	1
6	0	2
7	1	1
8	0	1
9	0	1
total :	32	19

Example 9.10: Distribution of pauses in ...*sofferte onde serene*....

The total duration of pauses (180”) is the sum of 32 short and 19 long pauses.

180 = 32 x + 19 x

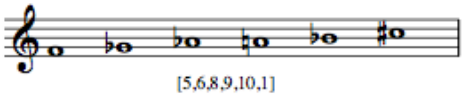
With this formula, the average duration of 2 to 3 seconds was determined for short pauses, and of 4 to 7 seconds for long pauses (Example 9.11).

= 2 - 3”
= 4 - 7”

Example 9.11: Average durations of pauses
in *Le sourire infini des ondes*.

9.3.5 Construction of the series

Le sourire infini des ondes is the first piece I composed with the CIG-technique that does not use the series of *Les racines du monde.* For the construction of the series I took the first chord of *...sofferte onde serene...* as a starting point. This first chord is based on a pitch class set (belonging to the set class) with forte number [6-14]. Reduced to its normal form, this pc-set is shown in Example 9.12. The prime form for this pc-set is (013458).



Example 9.12: pc-set of the first chord
of *...sofferte onde serene...*

My aim was to construct a series that contained permutations of pc-sets belonging to set class [6-14] as often as possible. In total, there are 720 possible orderings (permutations) of [6-14]. However, most of these cannot be used in the construction of the series because they do not consist entirely of CIG-3's, and therefore cannot be contained in a CIG-3-series.

An analysis of the prime form (013458) shows that its permutations contain CIG-3's that are all permutations of the following (unordered) pitch class sets:

(0,1,3)	(1,3,4)
(0,1,4)	(1,4,5)
(0,1,5)	(3,4,5)
(0,1,8)	(3,4,8)
(0,3,4)	(4,5,8)
(0,4,5)	

Eight of these can be paired because they are transpositions of the same pc-set; they contain the same interval sequence: (0,1,4) is an instance of the same pc-set as (4,5,8), (0,1,5) the same as (3,4,8), (0,1,8) the same as (0,4,5), and (0,3,4) the same as (1,4,5).

Therefore, finally only seven unordered pc-sets remain. Each of those can be ordered in six ways to form 42 CIG-3's. The interval content of the CIG-3's is listed below.

for	(0,1,3):	:	+1+2	+3-2	-1+3	+2-3	-3+1	-2-1
	(0,1,4) or (4,5,8)	:	+1+3	+4-3	-1+4	+3-4	-4+1	-3-1
	(0,1,5) or (3,4,8)	:	+1+4	+5-4	-1+5	+4-5	-5+1	-4-1
	(0,1,8) or (0,4,5)	:	+1-5	-4+5	-1-4	-5+4	+4+1	+5-1
	(0,3,4) or (1,4,5)	:	+3+1	+4-1	-3+4	+1-4	-4+3	-1-3
	(1,3,4)	:	+2+1	+3-1	-2+3	+1-3	-3+2	-1-2
	(3,4,5)	:	+1+1	+2-1	-1+2	+1-2	-2+1	-1-1

Each ordered appearance of [6-14] in the series of *Le sourire infini des ondes* should consist only of CIG-3's. There are 56 permutations (out of 720) of the prime form of [6-14] in which all groups of

three consecutive pitch classes (each group of two consecutive intervals) form one of the 42 possible CIG-3's. These 56 permutations are shown below (without brackets):

013458	013485	015438	015483	031458	051438
103458	103485	105438	105438	105483	130458
150438	301458	301548	310458	310548	345018
345108	354018	354108	384501	384510	501348
501438	510348	510438	534018	534108	543018
543108	584301	584310	801345	801435	801453
801543	810345	810435	810453	810543	834015
834051	834105	834150	834501	834510	843015
843105	845013	845103	854013	854031	854103
854130	854301	854310			

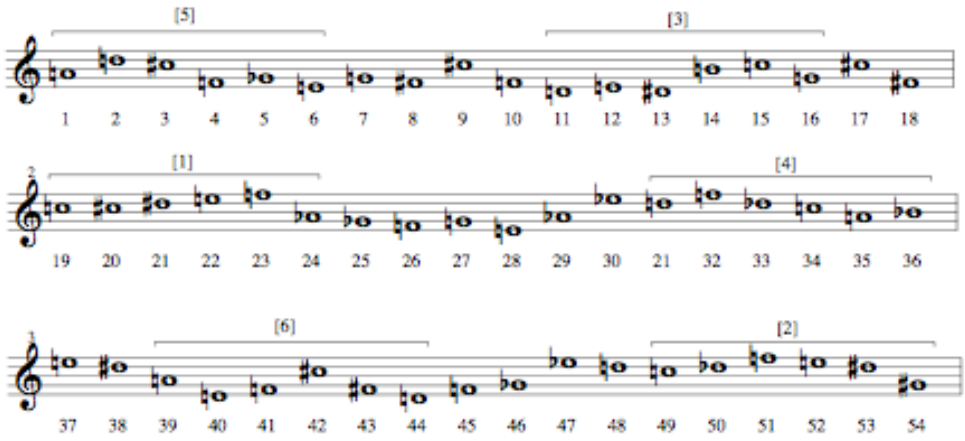
It is obvious that not all of these permutations of [6-14] can occur together in the series, because one and the same CIG-3 would occur more than once, and this is forbidden in CIG-3-series, even if the CIG-3's occur in different transpositions. (013458), for instance, cannot be used in the series together with (013485), because both ordered pc-sets contain CIG-3 (0,1,3).

Starting from the prime form (013458) considered as an ordered pc-set, all permutations of [6-14] that contain a repetition of the CIG-3's in the permutation (013458) were eliminated. This resulted in the six remaining permutations that could occur in the same CIG-3 series listed below, together with their ordered interval class content (Example 9.13).

[1]	(013458)	+1	+2	+1	+1	+3
[2]	(015438)	+1	+4	-1	-1	+5
[3]	(354018)	+2	-1	-4	+1	-5
[4]	(584301)	+3	-4	-1	-3	+1
[5]	(810453)	+5	-1	+4	+1	-2
[6]	(834051)	-5	+1	-4	+5	-4

Example 9.13: Six permutations of (013458) that can occur in the same CIG-3 series.

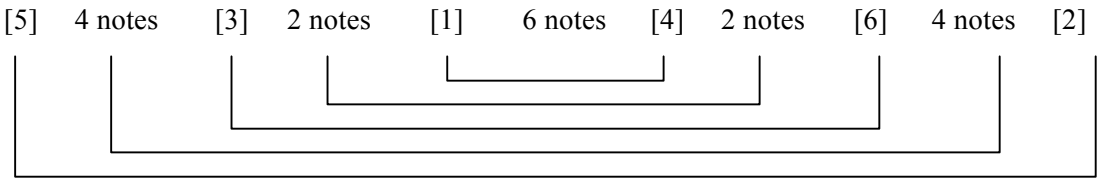
These six permutations were incorporated into the CIG-3-series of *Le sourire infini des ondes*. I constructed the sequence in such a manner that the six permutations of [6-14] are distributed symmetrically within the series, and that the four instances of ic 6 (the only ic that does not occur in the interval vector of [6-14]) are placed symmetrically around the two axes of symmetry. This resulted in the series shown in Example 9.14.



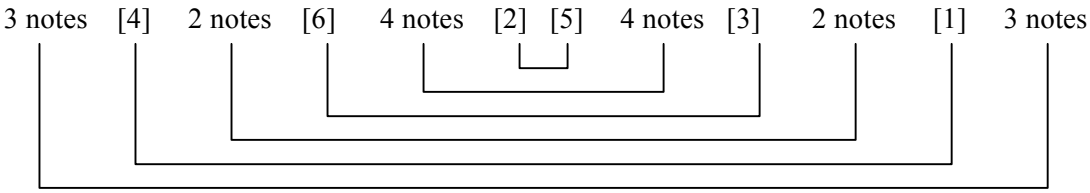
Example 9.14: The series of *Le sourire infini des ondes.*
Permutations of [6-14] are indicated with horizontal square brackets.

The six permutations of [6-14] in the series are indicated with numbers in square brackets. The axes of symmetry occur between notes 54 and 1 (the 'end' and 'beginning'[560] of series), and between notes 27 and 28 (the 'middle' of the series). The six permutations of [6-14] are distributed at equal distances from the axes of symmetry as shown below:

Axis of symmetry in the 'middle' of the series (between note 27 and 28):



Axis of symmetry at the beginning of the series (between note 54 and 1):



The ic 6's constitute the transitional intervals between notes 16 and 17, 18 and 19, 36 and 37 and 38 and 39, which are also situated symmetrically (at paired equal distances) from the axes of symmetry.

9.3.6 Determination of rhythmic cells

Once the series is determined, the next step in the compositional process consists of attributing rhythmic cells to every series note. In the case of *Le sourire infini des ondes,* this is done by defining nine rhythmic cells based on the interval content of the series. The complete set of unordered interval class numbers of the series of *Le sourire infini des ondes* is:

[560] 'End', 'beginning' and 'middle' are here written between quotation marks, because, strictly speaking, a CIG-series has no beginning, middle or end. It has a ring shape, as was explained in Chapter 1.

5 1 4 1 2 3 1 5 4 3 2 1 4 1 5 6 5 6 1 2 1 1 3 2 1 2 3

4 5 1 3 4 1 3 1 6 1 6 5 1 4 5 4 3 1 3 1 2 1 4 1 1 5 1

The square brackets in this string of numbers indicate the interval class numbers of the six permutations of [6-14]. Six of the rhythmic cells are determined by the interval content of those six permutations. The other rhythmic cells are formed by the interval class numbers between the six permutations in the series. The sets containing ic 6 are left out, because they would result in rhythmic cells with only three note lengths. The omission of ic 6 in the construction of rhythmic cells reflects the absence of ic 6 in [6-14] (and in the first chord of *...sofferte onde serene...*). The transition between the end and the beginning of the interval content series was also omitted because it only contains one number (which would have resulted in a rhythmic cell with only one note length).

This results in the following number strings for the nine rhythmic cells of *Le sourire infini des ondes*:

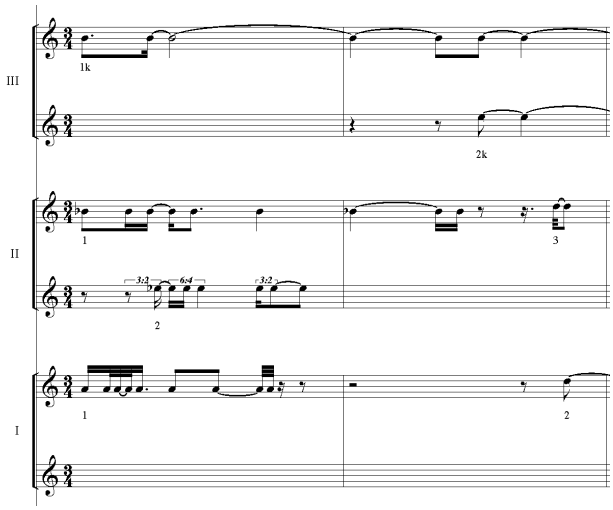
(1)	5	1	4	1	2		
(2)	3	1	5	4	3		
(3)	2	1	4	1	5		
(4)	1	2	1	1	3		
(5)	2	1	2	3	4	5	1
(6)	3	4	1	3	1		
(7)	5	1	4	5	4		
(8)	3	1	3	1	2		
(9)	1	4	1	1	5		

These values correspond to the note lengths in the rhythmic chart shown in Example 1.14. Value 1 corresponds to the length unit of each augmentation (demi-semiquaver, triplet semiquaver, semiquaver, triplet quaver, quaver, dotted quaver). Value 2 is twice the length unit, value 3 three times the length unit, etc. (see Chapter 1).

9.3.7 Determination of the forms of the series used

In order to obtain a higher degree of dissonance, three transpositions of the series a semitone apart are used simultaneously in the piece. Series I (transposition I) is the original un-transposed series (starting on A). Series II begins a semitone higher (on B flat instead of A), and series III another semitone higher (starting on B). The first beats of the combined series in the RHS of *Le sourire infini des ondes* is shown in Example 9.15.

Although the series are used simultaneously, different rhythmic cells are assigned to the notes of each of the transposition of the series in order to obtain a certain rhythmic independence of the three transpositions of the series. In addition, the start of the rhythmic cells coincides only at the beginning of each of the nine sections.



Example 9.15: Beginning of the RHS of *Le sourire infini des ondes*,
showing the original series (I) and two semitone transpositions (II and III) used simultaneously.

The entire piece consists of 618 quaver beats. A musically useful and balanced density of the rhythmic cells is obtained by running through each transposition of the series three times forming three segments (Segments A, B and C). This results in a total of 3 x 3 x 54 = 486 rhythmic cells for the entire piece (three times each of the three forms of the series, each containing 54 notes).

The series are attached in a symmetrical manner to the three segments in the following way (Example 9.16):

segment	A	B	C
section	1 - 4	5 - 6	7 - 9
series notes	1 → 54	27 → 1 / 28 → 54	54 → 1

Example 9.16: Distribution of series notes over the three segments of *Le sourire infini des ondes.*

In Segment A, the series is used in prime form. Segment C uses the retrograde of the series and Segment B is based on a combination of both.

The series notes were distributed over the segments and sections as shown in Example 9.17.

segment	**section**	**number of beats**	**number of series notes**	**series notes**
A	1	39	12	1 – 12
	2	39	12	13 – 24
	3	20	6	25 – 30
	4	95	24	31 - 54
B	5	96	27	27 – 1
	6	112	27	28 - 54
C	7	144	36	54 – 19
	8	56	14	18 – 5
	9	16	4	4 - 1

Example 9.17: Distribution of series notes over the three segments
and nine sections of *Le sourire infini des ondes*.

9.3.8 Construction of the N-matrix

The next step in the composition process consists of the construction of an N-matrix[561] for the series, a process that had its for-runner in the distribution of rhythmic cells over the series notes in *Les racines du monde* and that was first implemented in its full version in *Après la pluie* for piano and live electronics (2008).

The N-matrix of *Le sourire infini des ondes* is a matrix consisting of nine rows and six columns that are determined by the interval class content of the CIG-3's. The prime forms of the nine chromatic pc-sets (see Example 1.5) are listed in 'ascending'[562] order below:

(0,1,2)
(0,1,3)
(0,1,4)
(0,1,5)
(0,1,6)
(0,5,6)
(0,4,5)
(0,3,4)
(0,2,3)

The 54 permutations of those chromatic pc-sets (six for each pc-set) are then listed in ascending order from left to right after the corresponding prime forms, resulting in the following matrix:

(0,1,2)	(0,2,1)	(1,0,2)	(1,2,0)	(2,0,1)	(2,1,0)
(0,1,3)	(0,3,1)	(1,0,3)	(1,3,0)	(3,0,1)	(3,1,0)
(0,1,4)	(0,4,1)	(1,0,4)	(1,4,0)	(4,0,1)	(4,1,0)
(0,1,5)	(0,5,1)	(1,0,5)	(1,5,0)	(5,0,1)	(5,1,0)
(0,1,6)	(0,6,1)	(1,0,6)	(1,6,0)	(6,0,1)	(6,1,0)
(0,5,6)	(0,6,5)	(5,0,6)	(5,6,0)	(6,0,5)	(6,5,0)
(0,4,5)	(0,5,4)	(4,0,5)	(4,5,0)	(5,0,4)	(5,4,0)
(0,3,4)	(0,4,3)	(3,0,4)	(3,4,0)	(4,0,3)	(4,3,0)
(0,2,3)	(0,3,2)	(2,0,3)	(2,3,0)	(3,0,2)	(3,2,0)

Next, each of the 54 ordered pc-sets is replaced by an interval class-set (ic-set). These ic-sets contain the interval content between the successive pitch classes in the ordered pc-sets; they are obtained by subtracting the successive numbers in the representations of the pc-sets between round brackets; the first number is subtracted from the second; the second from the third. The ic-set for pc-set (0,1,4), for instance, is obtained by subtracting 0 from 1 (1 – 0 = 1) and 1 from 4 (4 – 1 = 3) resulting in ic-set (+1,+3). All the results are listed in the matrix shown in Example 9.18 below[563].

1 1	2 -1	-1 2	1 -2	-2 1	-1 -1
1 2	3 -2	-1 3	2 -3	-3 1	-2 -1
1 3	4 -3	-1 4	3 -4	-4 1	-3 -1
1 4	5 -4	-1 5	4 -5	-5 1	-4 -1
1 5	6 -5	-1 6	5 -6	-6 1	-5 -1
5 1	6 -1	-5 6	1 -6	-6 5	-1 -5
4 1	5 -1	-4 5	1 -5	-5 4	-1 -4
3 1	4 -1	-3 4	1 -4	-4 3	-1 -3
2 1	3 -1	-2 3	1 -3	-3 2	-1 -2

Example 9.18: Distribution of ic-sets in the matrix of *Le sourire infini des ondes.*

[561] N stands for note number (referring to the position of the notes within the series).
[562] The term 'ascending' has to be understood here as follows: it means the 3-digit numbers *that would be obtained by removing the commas in the representations of the pc-sets* are put in ascending order. The 3-digit number for (0,1,2) for instance is 012.
[563] In this list, the round brackets for the ic-sets are omitted. Plus (+) signs are also omitted. Note also that there is no essential difference between (+)6 and -6.

Next, the ic-sets in the matrix are replaced by the number of the series note that is the central pitch class in the CIG-3 corresponding to the ic-set. Series note 22 in the series of *Le sourire infini des ondes*, for instance (E, see Example 9.14) is the central pitch class in the CIG-3 representing ic-set (+1,+1), the ic-set in the top left corner of the matrix. This procedure results in the N-matrix shown in Example 9.19.

22	12	26	5	49	52
20	24	31	27	35	25
23	10	3	32	14	47
50	43	53	29	40	33
1	39	38	18	19	30
54	37	16	36	17	8
4	2	42	15	9	13
45	51	28	41	44	34
21	7	6	46	11	48

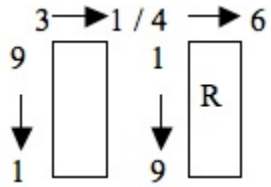
Example 9.19: N-matrix for *Le sourire infini des ondes.*

9.3.9 Attachment of rhythmic cells and augmentations to series notes

Using the N-matrix, rhythmic cells and augmentations were then ascribed to every series note in the following manner: the six columns of the N-matrix correspond to the six augmentations of the rhythm chart; the nine rows correspond to the nine rhythmic cells that were determined for the piece. The ascription differs from segment to segment, and from series transposition to series transposition. It was determined according to the schedule shown in Example 9.21. Segment B was divided into the B1 and B2, respectively, corresponding to section 5 (retrograde of the series, notes 27 - 1) and section 6 (prime form of the series, notes 28 - 54).

The squares in the diagram represent the N-matrix or two halves[564] of the N-matrix. The letter "R" in the squares indicates that retrogrades of the rhythmic cells are used. To understand how to read the diagrams in detail, let's have a closer look at the second transposition (II) in segment A, for instance. The N-matrix was attributed as shown in Example 9.20 below. Columns 1 to 3 of the matrix correspond to augmentations 3 to 1 (3 → 1). Rows 1 to 9 in the matrix half of these columns correspond to rhythmic cells 9 to 1 (9 → 1) respectively. Likewise, columns 4 to 6 of the N-matrix correspond to augmentations 4 to 6 (4 → 6) and rows 1 to 9 of this matrix half are ascribed rhythmic cells 1 to 9 (1 → 9). In this second matrix half, the rhythmic cells occur in retrograde (indicated with the letter R).

[564] The vertical division of the matrix in two (columns 1 to 3, and columns 4 to 6) reflects the division of segment B in two halves (segments B1 and B2).



Example 9.20: Distribution of augmentations and rhythmic cells
in sections of N-matrix of *Le sourire infini des ondes.*

	transposition I	transposition II	transposition III
segment A	1 → 6 1 ↓ 9	3 → 1 / 4 → 6 9 ↓ 1 R 1 ↓ 9	6 → 1 9 ↓ 1 R
segment B1	6 → 1 1 ↓ 9 R	1 → 3 / 6 → 4 9 ↓ 1 R 1 ↓ 9	1 → 6 9 ↓ 1
segment B2	1 → 6 9 ↓ 1 R	6 → 4 / 1 → 3 9 ↓ 1 1 ↓ 9 R	6 → 1 1 ↓ 9
segment C	6 → 1 9 ↓ 1	4 → 6 / 3 → 1 9 ↓ 1 R 1 ↓ 9	1 → 6 1 ↓ 9 R

Example 9.21: Distribution of rhythmic cells and transpositions in *Le sourire infini des ondes.*

9.3.10 Determination of the distance between beginnings of rhythmic cells in the RHS

After a rhythmic cell was determined for each series note in the three transpositions of the series and each of the three segments, a RHS was constructed by determining the distance (in time) between the start of successive rhythmic cells. In each of the nine sections of the piece, the rhythmic cell of the first note of each transposition starts at the same moment (as can be seen in the excerpt of the RHS in Example 9.16). The starting position of every rhythmic cell within the RHS is determined in such a manner that the rhythmic cells fill the entire length of the section proportionately. The 'proportioned' distance between the start of the cells is based on the unordered interval class content of the series, as can be seen in the columns 'distance from previous cell' in the diagram in Example 9.22 below, showing the distribution of distances between entrances of rhythmic cells for section 1 of *Le sourire infini des ondes*. This resulted in the RHS of which an excerpt is shown in Example 1.17a, and which was the starting point for the composition of the score.

SECTION 1	I		II		III	
Note number	Distance from previous cell	Distance from beginning of section	Distance from previous cell	Distance from beginning of section	Distance from previous cell	Distance from beginning of section
1	**0**	**0**	**0**	**0**	**0**	**0**
2	**5+1**	**6**	**1**	**1**	**5**	**5**
3	**1**	**7**	**1+4**	**6**	**4**	**9**
4	**4**	**11**	**4+1**	**11**	**1**	**10**
5	**1**	**12**	**2**	**13**	**1+2**	**13**
6	**2+3**	**17**	**2**	**15**	**3**	**16**
7	**1**	**18**	**3+1**	**19**	**3**	**19**
8	**1+5**	**24**	**5**	**24**	**1**	**20**
9	**5**	**29**	**4**	**28**	**5+4**	**29**
10	**3**	**32**	**4+3**	**35**	**4**	**33**
11	**3+2**	**37**	**2**	**37**	**3**	**36**
12	**1**	**38**	**2**	**39**	**2+1**	**39**

Example 9.22: Determination of the distance between entrances
of rhythmic cells in the RHS for section 1 of *Le sourire infini des ondes.*

9.4 *Danse du feu*

9.4.1 Introduction

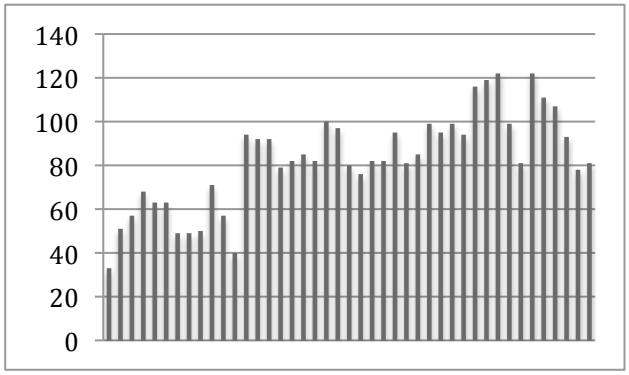
Danse du feu is composed in the course of my research on consonance and tonality, at a stage where it became clear to me that interval groups of higher order (more than three pitch classes) have to be taken into consideration when a high degree of atonality and dissonance is aspired. The structural idea behind both *Danse du feu* and *Un souffle de l'air que respirait le passé* is based on experiments with the possibilities of using tetrachords with a high degree of atonality and dissonance. During these experiments it occurred to me—as a result of sheer serendipity— that chromatic interval groups with only two different pitch classes (3-note CIG-2's) were also possible in the construction of CIG-series, resulting in series with 56 notes (see Section 5.4.5.2 and Example 5.28).

9.4.2 Chromatic pc-sets of order 4

The pc-sets with cardinality 4 that have 'the lowest' degree of tonality and prime consonance are sets belonging to set classes [4-1] through [4-9] as can be seen in Example 5.4; these are the chromatic pc-sets of order 4 (see Section 5.3)[565] and are the basis of CIG-4's. The degree of consonance of the chromatic pc-sets of order 4 is lower than that of all the other tetrachords. Five of the other tetrachord set classes ([4-z15i], [4-18], [4-19i], [4-25] and [4-z29]) have a degree of tonality that is comparable to that of the group formed by set classes [4-1] through [4-9], but their degree of prime consonance is higher, and therefore the combination of their degree of tonality and prime consonance discards them from the group of 'lowest' degree of tonality and prime consonance (the sum of their degree of

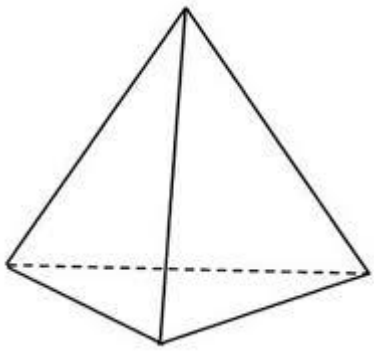
[565] I initially called them 'low 4-sets' due to the fact that they have the lowest degrees of tonality and prime consonance within their cardinality group.

tonality and consonance is higher than that of the group of chromatic pc-sets of order 4, as can been seen in Example 9.23).



Example 9.23: Sum of degree of tonality and prime consonance of set classes of cardinality 4.
The first 12 ([4-1] through [4-9]) have a clearly lower value
than all other classes within the cardinality group.

Danse du feu and *Un souffle de l'air que respirait le passé* are based on a series consisting exclusively of CIG-4's, each consisting of two CIG-3's. They are therefore examples of 3/4-serialism. In ancient Greece the tetrahedron (see Example 9.24) represented the Empedoclian element of fire.[566] A tetrahedron is a three dimensional geometric figure with four triangular faces, which coincidentally reflects the combination of numbers 3 and 4 in 3/4 serialism in the 'dance of *fire'* (*Danse du feu*).



Example 9.24: Tetrahedron symbolizing the element of Fire in ancient Greece.

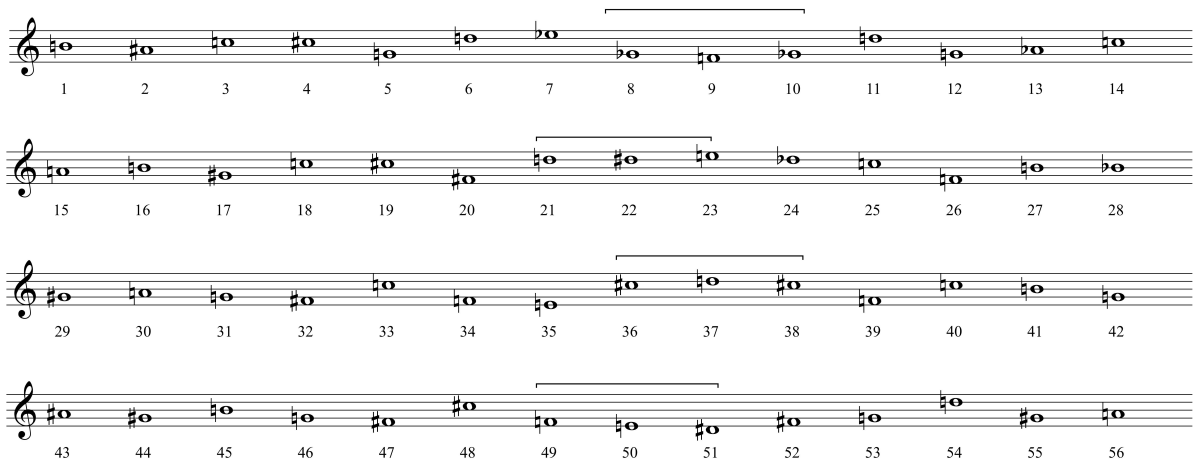
9.4.3 Construction of the series

In a first instance, I tried to construct a series in which every possible permutation of the chromatic pc-sets of order 4 (every possible CIG-4) occurs exactly once. Such an all-CIG-4 series would consist of 288 pitch classes. It would be so long that using them once would probably be sufficient to write whole 12 to 15-minute pieces, such as *Danse du feu*. However, not all groups of three successive notes within the CIG-4's are CIG-3's. [4-6], for instance, has permutations that contain an instance of [3-9], which is not a CIG. Likewise, CIG's belonging to set classes [4-5] and [4-5i] contain instances of [3-8] and [3-8i]; permutations of [4-4] and [4-4i] contain instances of [3-7] and [3-7i]; [4-2] and [4-2i] contain [3-6]. Even if those permutations are left out (84 in total) it still leaves 204 interval groups of 4 elements in the series. Since there are only 12 different permutations of [4-9] (six of the permutations are identical to six others), this leaves 'only' 192 different interval groups in the series, which is still impractically much.

[566] See: John R. Pierce, *The Science of Musical Sound*, Scientific American Books, revised edition, 1992, p. 20.

Furthermore, although every CIG-4 would occur exactly once in the series, it would not be amotivic, since many CIG-3's would occur more than once. But even then, the construction of a closed all-CIG-4 series proved to be impossible.[567] Therefore the objective to construct an all-CIG-4 series was abandoned altogether. Instead, the series of *Danse du feu* is constructed on the basis of all CIG-3's (the 'classical' way) with the addition the four 3-note CIG-2's in such a way that every two consecutive CIG-3's form a CIG-4, to form a 56-note 3/4-CIG series.

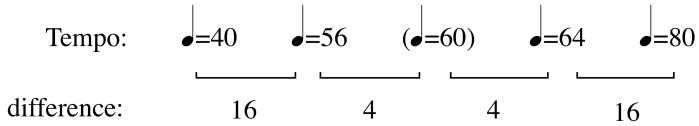
The series was constructed in such a way that its second half (notes 29 to 56) is the inversion of the first half (notes 1 to 28). In addition, the sections within the series apart from the 3-note CIG-2's are each others retrograde inversions (in transposition): notes 3 to 8 and 56 to 51, notes 10 to 21 and 49 to 38; notes 23 to 36 are their own transposed retrograde inversion. Moreover, the first eight notes of the series form a group of which notes 23 to 30 are a transposed retrograde; likewise notes 29 to 36 are the inversion and note 51 to 2 are a transposed retrograde. Notes 10 to 16 form a group of seven notes of which notes 15 to 21 are a transposed inversion of the retrograde, notes 38 to 44 are a transposed inversion and notes 43 to 49 are a transposed retrograde. The resulting series is shown in Example 9.25.



Example 9.25: The series of *Danse du feu.*
The four 3-note CIG-2's are indicated with square brackets.

9.4.4 Determination of tempo and number of series used

Danse du feu has four tempi: 40, 56, 64 and 80 crotchet beats per minute. The four tempi are distributed symmetrically around the one-second-per-beat absent tempo (60), as follows (Example 9.26):

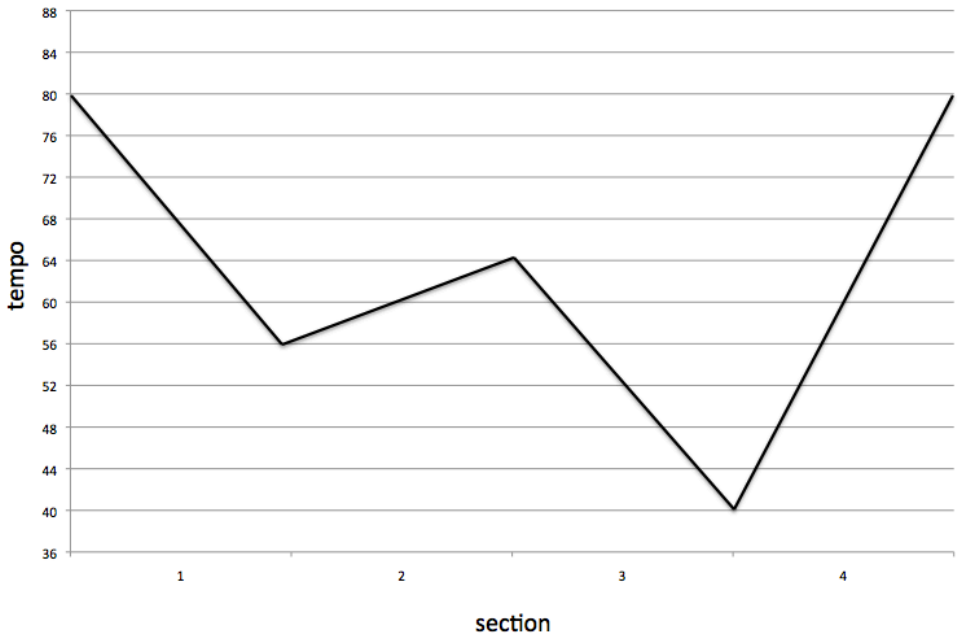


Example 9.26: distribution of the four tempi of *Danse du feu*
around the absent tempo of 60 crotchet beats per minute.

[567] This may be caused by the fact that, in order for all CIG-4 series to be possible, there have to be as many CIG-3's of any type at the beginning of the CIG-4's as there are at the end; an aspect that was not further assessed in the present research.

The tempo of *Danse du feu* changes continuously. The piece starts at 80 crotchet beats per minute and immediately but imperceptibly slows down to 56 beats per minute at the start of the second section (Tempo 2°, bar 73), only to speed up to 64 beats per minute towards the beginning of Section 3 (Tempo 3°, bar 125). From this point, the piece gradually slows down again to 40 beats per minute (Tempo 4°, bar 188), where the final accelerando towards 80 beats per minute in the last bar of the piece starts (see the graph in Example 9.27). The end tempo is the same as the initial tempo in order to create a 'closed' tempo structure, which reflects the 'closed' structure of the series.

The average tempo is 60. This means that, in order to obtain a piece lasting ca. 12 minutes it should contain approximately 720 beats. 784 beats, which is 28x28, or 56x14 comes relatively 'close'. A suitable 'density' of rhythmic cells can be obtained by using the series four times in the whole piece and using four simultaneous series (or forms of the series). In total this would yield 4 x 4 x 56 = 896 rhythmic cells. This seems to be too much. Using three simultaneous series results in 672 cells. Compared to the ca. 540 cells in *Danse de la terre* (with its 720 beats) is this still much, but reasonable. Therefore I opted for 672 rhythmic cells for 784 beats, and three simultaneous series, henceforth called Series 1, II and III, each repeated four times (in four sections, Sections A, B, C and D). Series I is the original series shown in Example 9.25. Series II is Series I transposed down a semitone; Series III is series I transposed up a semitone.



Example 9.27: Symmetrical structure of continuously changing tempo in *Danse du feu.*

9.4.5 Determination of rhythmic cells

The RHS of *Danse du feu* is constructed with eight rhythmic cells, each consisting of seven note lengths, and in seven augmentations. The first four rhythmic cells (cells 1 to 4) are determined by the unordered interval content between the first 29 successive series notes in groups of seven:

(1) 1 2 1 6 5 1 3
(2) 1 1 4 5 1 4 3
(3) 2 3 4 1 5 4 1
(4) 1 3 1 5 6 1 2

Note that the sums of the seven numbers for the rhythmic cells are almost identical (19 or 20).

Since the second half of the series is the inversion of the first half, the unordered interval content between notes 29 and 1 is the same as that of the first half. That yields exactly the same rhythmic cells. Therefore the numbers that determine the rhythmic cells are 'mirrored": 1 and 6 are swapped (1 becomes 6 and 6 becomes 1), 2 and 5, and 3 and 4 in the first four rhythmic cells, resulting in the following four additional rhythmic cells:

(5) 6 5 6 1 2 6 4
(6) 6 6 3 2 6 3 4
(7) 5 4 3 6 2 3 6
(8) 6 4 6 2 1 6 5

The 'lengths' of these four cells (cells 5 to 8) is considerably longer than that of the first four (the sums of the numbers are 29 and 30).

Each of these eight rhythmic cells is used in seven augmentations in the RHS of *Danse du feu*, instead of the usual six (see rhythm chart in Example 1.14). In addition to the six basic units in the usual rhythmic chart, the unit of triplet crotchet is added between the fifth and sixth augmentation unit (quaver and dotted quaver).

9.4.6 Construction of the N-matrix

As for the composition of *Le sourire infini des ondes,* an N-matrix was used in the composition of *Danse du feu*. In contrast to *Le sourire infini des ondes* (see Example 9.19), a 8 by 7 matrix was necessary instead of 9 by 6, as a result of the eight rhythmic cells in seven augmentations (instead of nine rhythmic cells in six augmentations). The distribution of CIG's in the matrix is shown in Example 9.28.

1 -1	**1 1**	**2 -1**	**-1 2**	**1 -2**	**-2 1**	**-1 -1**
1 2	**3 -2**	**-1 3**	**2 -3**	**-3 1**	**-2 -1**	**1 3**
4 -3	**-1 4**	**3 -4**	**-4 1**	**-3 -1**	**1 4**	**5 -4**
-1 5	**4 -5**	**-5 1**	**-4 -1**	**1 5**	**6 -5**	**-1 6**
5 6	**6 1**	**-5 -1**	**5 1**	**6 -1**	**-5 6**	**1 6**
6 5	**-1 -5**	**4 1**	**5 -1**	**-4 5**	**1 -5**	**-5 4**
-1 -4	**3 1**	**4 -1**	**-3 4**	**1 -4**	**-4 3**	**-1 -3**
2 1	**3 -1**	**-2 3**	**1 -3**	**-3 2**	**-1 -2**	**-1 1**

Example 9.28: Distribution of CIG's (represented as ordered ic-sets) in the N-matrix of *Danse du feu.*

Just like in the N-matrix of *Le sourire infini des ondes*, for series I in *Danse du feu* the ic-sets in the matrix are replaced by the number of the series note that is the central pitch class in the CIG-3 corresponding to the ic-set. This yields the N-matrix shown in Example 9.29.

37	22	1	2	30	29	50
56	43	51	16	36	31	7
14	38	45	21	24	13	20
25	39	6	46	19	5	32
26	55	40	12	27	54	4
33	47	18	34	11	53	48
41	52	49	17	10	42	35
3	8	44	23	15	28	9

Example 9.29: N-matrix for series I of *Danse du feu.*

The N-matrix for Series II is obtained by swapping the position of rows 1 through 4 and rows 5 through 8 in the N-matrix for series I. The resulting N-matrix is shown in Example 9.30.

26	55	40	12	27	54	4
33	47	18	34	11	53	48
41	52	49	17	10	42	35
3	8	44	23	15	28	9
37	22	1	2	30	29	50
56	43	51	16	36	31	7
14	38	45	21	24	13	20
25	39	6	46	19	5	32

Example 9.30: N-matrix for series II of *Danse du feu.*

For the N-matrix of series III, the positions of columns 1 through 3 and 5 through 7 of the N-matrix for Series II are swapped. Only column 4 remains in its previous position. The result is shown in Example 9.31.

27	54	4	12	26	55	40
11	53	48	34	33	47	18
10	42	35	17	41	52	49
15	28	9	23	3	8	44
30	29	50	2	37	22	1
36	31	7	16	56	43	51
24	13	20	21	14	38	45
19	5	32	46	25	39	6

Example 9.31: N-matrix for series III of *Danse du feu.*

9.4.7 Attachment of rhythmic cells and augmentations to the N-matrix

The eight rhythmic cells and their seven augmentations are attached to the N-matrix of *Danse du feu* the following way: for all transpositions of all series (Series I through III) in the first section (Section A), rhythmic cells 1 through 8 are distributed in order over the eight rows of the respective N-matrices (rhythmic cell 1 on row 1, rhythmic cell 2 on row 2, …). In like manner, the augmentations are attached to the seven columns of the N-matrices: augmentation 1 to the first column of the three N-matrices, augmentation 2 to the second, and so on.

In the three following sections of the piece, the procedure is kept the same; only the order of rhythmic cells and augmentation is changed. In Section B the order of the augmentations is reversed (augmentation 7 to 1 are attached in descending numerical order to columns 1 to 7); in Section C the order of the rhythmic cells is reversed, and in section D both the order of rhythmic cells and augmentations, as can be seen in Example 9.32.

	All series
Section A	1 → 7 1 ↓ 8
Section B	7 → 1 1 ↓ 8
Section C	1 → 7 8 ↓ 1
Section D	7 → 1 8 ↓ 1

Example 9.32: Attachment of 8 rhythmic cells (1 → 8)
and 7 augmentations (1→ 7) to the N-matrices (represented by squares)
for the three series in sections A to D of *Danse du feu.*

9.4.8 Determination of the distance between beginnings of rhythmic cells in the RHS

The last step in the construction of the RHS of *Danse du feu* is the determination of the distance in time between the beginning of the rhythmic cells. Within each section, the procedure is similar to the one used in *Le sourire infini des ondes* (see Section 9.3.10). Each of the four sections of the RHS is constructed separately. In sections A and C, the three series start together, in sections B and D, they end together.

For section A and C:

Rhythmic cell of note 2 of series 1 enters $| i^{before}_{2} | + | i^{after}_{2} |$ units after note 1
Rhythmic cell of note 2 of series 2 enters $| i^{before}_{2} |$ units after note 1
Rhythmic cell of note 2 of series 3 enters $| i^{after}_{2} |$ units after note 1

The series with the shortest total length at the beginning of the next rhythmic cell (n) then adds $|i^{before}_{n}| + |i^{after}_{n}|$ units for the next entrance, the 'longest' series adds the minimum of $|i^{before}_{n}|$ and $|i^{after}_{n}|$, the other remaining series adds the maximum of $|i^{before}_{n}|$ and $|i^{after}_{n}|$.

This procedure is repeated until the end of the series. Whenever there is a two-way tie, my aim was to 'vary as much as possible' (e.g. if the maximum was added to series I when the tie is reached, the minimum was added next). In a three way tie: $| i^{before}_{2} | + | i^{after}_{2} |$ units were added to the series that didn't add $| i^{before}_{2} | + | i^{after}_{2} |$ for the longest time.

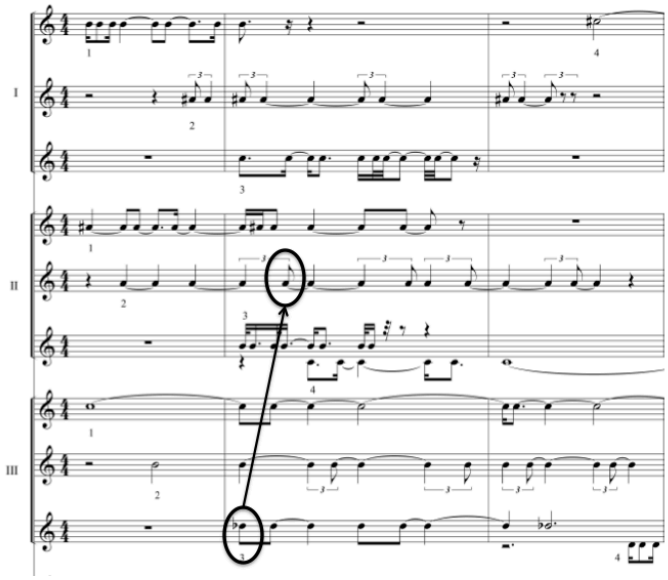
For section B and D, this procedure is reversed (starting from note 56).

9.4.9 Increasing melodic movement

The beginning and end of *Danse du feu* are highly energetic. At the end, this effect is achieved by the loud *tutti* with quickly repeated notes. In the energetic opening I wanted to create fast melodic runs. In order to do that, the notes in the RHS are played as single short (semiquaver) notes—only the attacks of the RHS notes are played—accompanied by two adjacent series notes. In addition, short melodic patterns are added. Those are determined as follows:

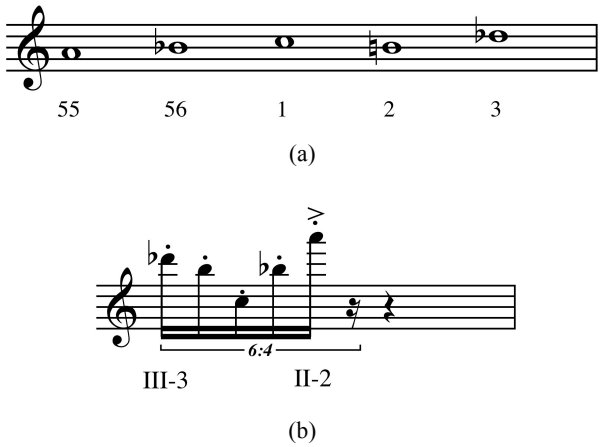
A note p of any version of the series in the RHS, is connected with a note r in a different version (transposition) of the series following that note. The time interval between notes p and r is filled in with (at least three) series notes of the same length (forward or backward (retrograde of series) starting with p and ending with r in the series of p, whenever this is possible (whenever there are notes p and r that fulfil the conditions, and whenever the time interval allows for it).

Consider for instance the third note in Series III (series note AIII-3, D flat, note p) and the second note of series note II (AII-2, A natural, note r) in Section A (see Example 9.33).



Example 9.33: Bar 1-3 of the RHS for Section A of *Danse du feu.*
Series notes AIII-3 (D flat) and AII-2 (A natural) are indicated.

The time interval between these two notes (belonging to different versions series, in casu series III and II) is 'filled in' with short notes of the same value (here five triplet semiquavers) belonging to series III. In this case, notes 55 to 3 of the retrograde of Series III fit. These are shown in Example 9.33 a. The time interval between notes AIII-3 and AII-2 is filled in with these notes in the first flute, the vibraphone, and the first violin parts in the score (the first flute part is shown in Example 9.33 b).



Example 9.33: a) Notes 55 to 3 of Series III for *Danse du feu.*
b) Excerpt from bar 2 of Flute 1 in *Danse du feu* based on the retrograde of this note group to form a melodic line between note III-3 and II-2.

This procedure is not customary in my style. *Danse du feu* is actually the first (and to date the only) piece in which I used it. It illustrates the possibilities to extend the idioms and styles that can be obtained with CIG-serialism. It proves that serialism is only a technique, and that it does not determine style, and (only to a limited extent) idiom.

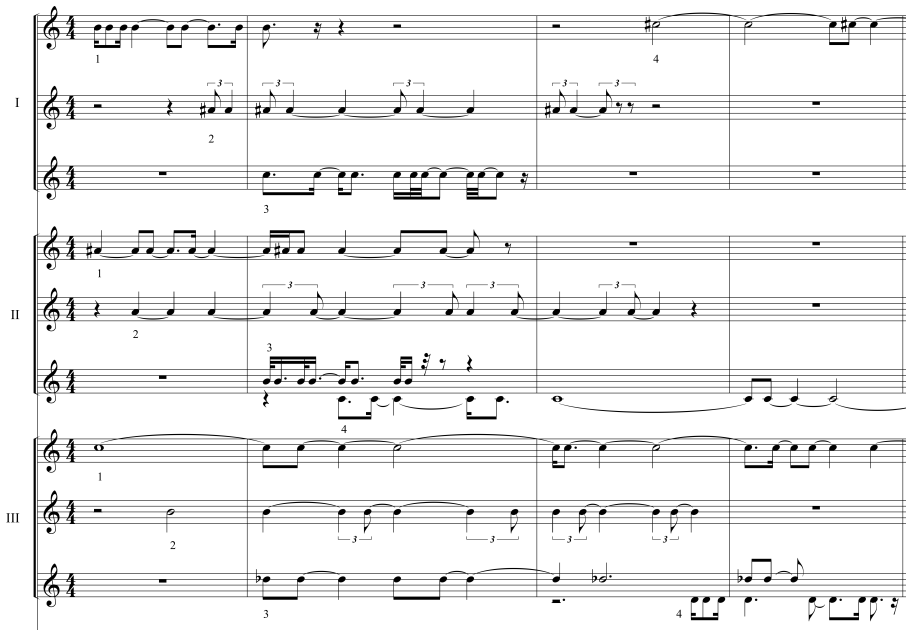
9.5 *Un souffle de l'air que respirait le passé*

In the process of composition, I normally adapt the construction of the RHS of a piece to the final sounding result (solo pieces or pieces for small ensemble have a 'simpler' RHS; orchestral pieces can have more complex combinations of the series and a more dense RHS).[568] In the case of *Danse du feu* and *Un souffle de l'air que respirait le passé,* however, I wanted to demonstrate the limited impact of a series on the final result, and to show that, even with an identical RHS, it is the creative role of the composer that determines the sounding result of the compositional process most, and not the strictly determined, highly confined series and RHS. Therefore I based *Danse du feu* and *Un souffle de l'air que respirait le passé* on one and the same series and RHS.

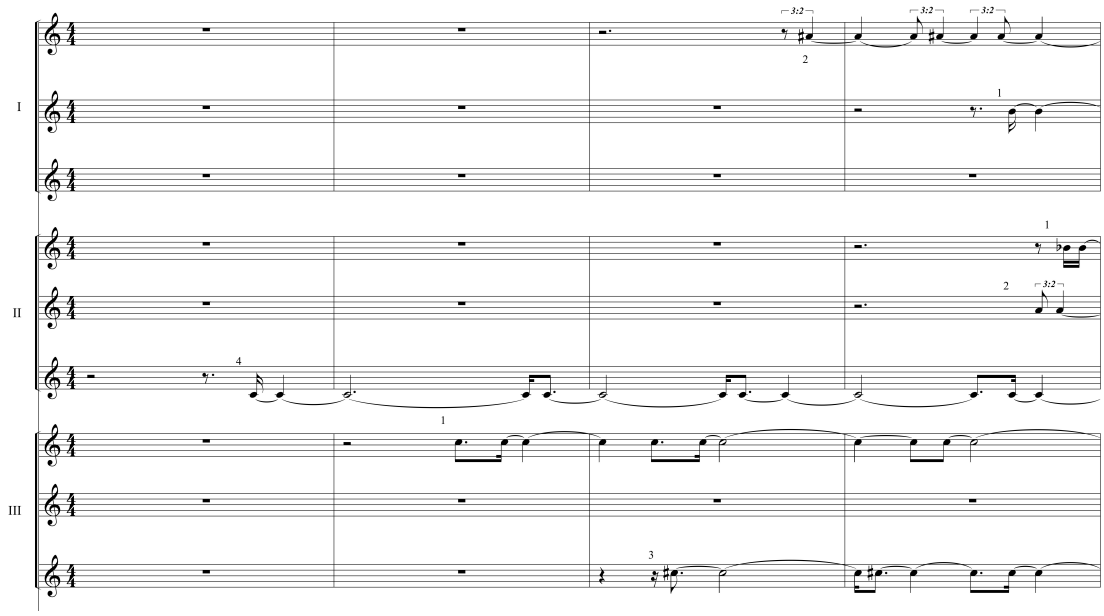
The RHS of both pieces consists of the same four sections (Section A to D), which are placed in the same order (A-B-C-D).[569] But whereas *Danse du feu* starts on Section A, *Un souffle de l'air que respirait le passé* begins with section D, followed by A, B and C in that order. This 'phase shift' occurs as a primary way of adapting the RHS to the envisaged sounding result. It shows the cyclic nature of the RHS, which reflects the 'closed' nature of CIG-series. The shift is done in complete artistic freedom. I could have chosen a change in the order of the sections in *Un souffle de l'air que respirait le passé* (A,C,D,B for instance), but this didn't seem necessary to obtain the desired result.

[568] See Section 1.2 on teleology.
[569] Note that Luigi Nono's *Prometeo* (the piece the title of *Un souffle de l'air que respirait le passé* was distilled from), like *...sofferte onde serene*…, has nine sections, not four.

By shifting the structure the density of the RHS corresponds better to the tension structure I pursued. Indeed, a very dense RHS such as the beginning of section A is better suited for music with a dense texture. Since I wanted *Danse du feu* to start loud and energetically, but *Un souffle de l'air que respirait le passé* very softly, it seemed better not to start the latter piece with section A but with section D and its transparent opening and sparse texture. The difference in density and texture of the RHS of Sections A and D is apparent in Examples 9.34 and 9.35, showing the first four bars of both.

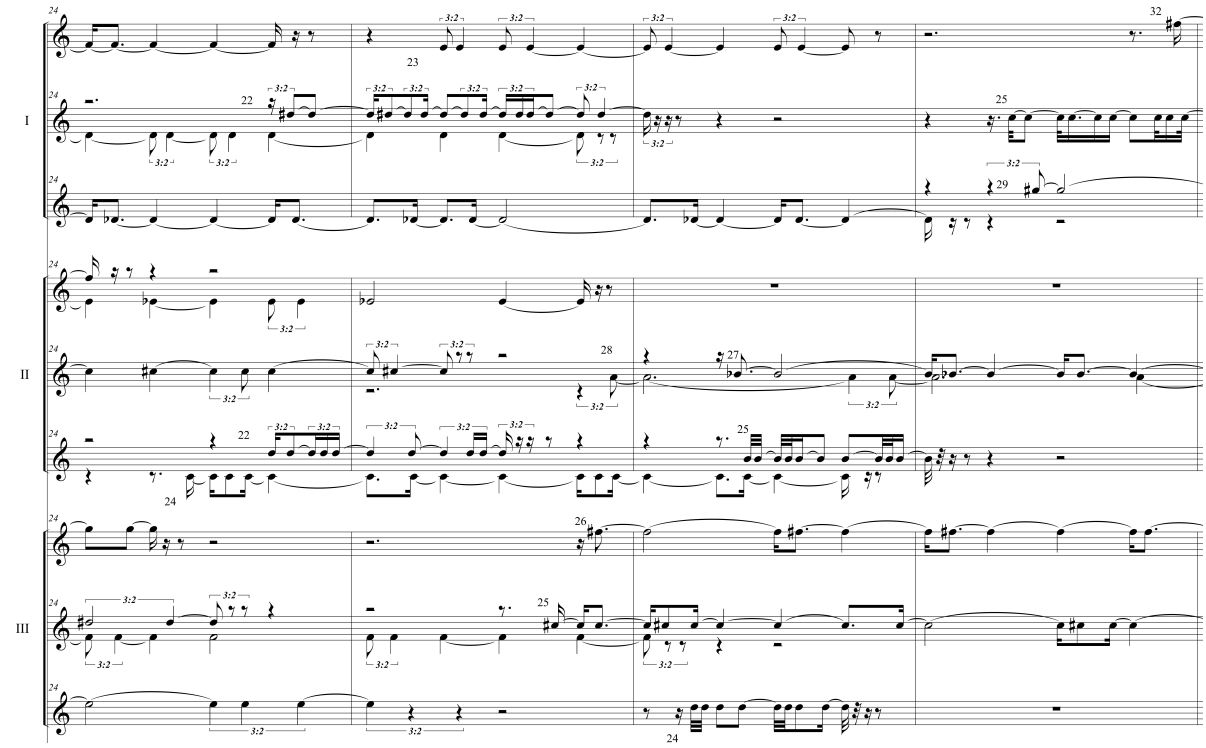


Example 9.34: Bar 1-4 of section A from the RHS of *Danse du feu*
(the second section in *Un souffle de l'air que respirait le passé*).

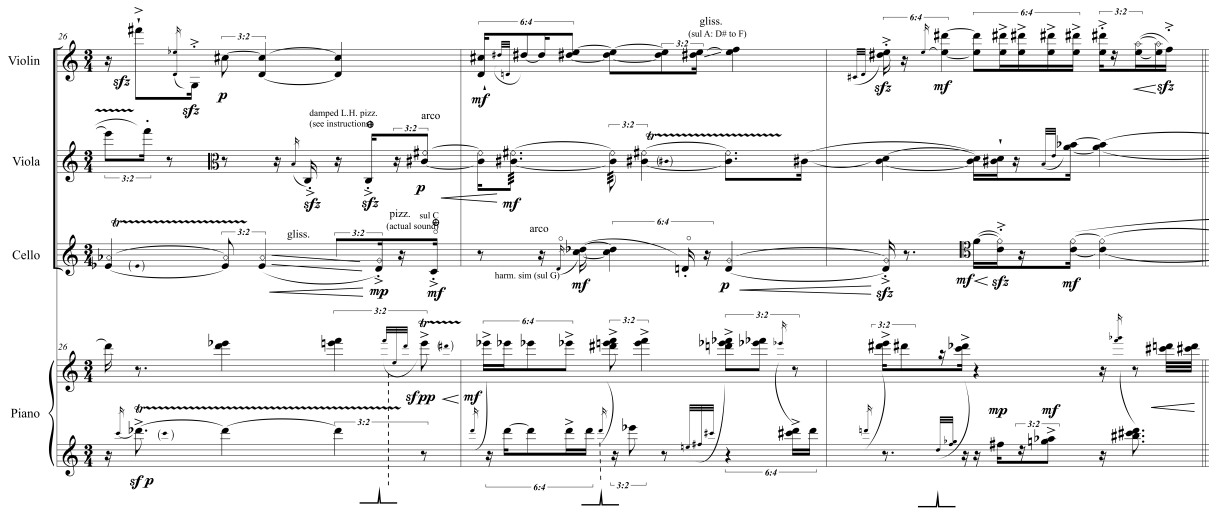


Example 9.35: Bar 1-4 of section D from the RHS of *Danse du feu*
(the first section in *Un souffle de l'air que respirait le passé*).

Thus, the determining role of the RHS is limited and adaptable to creative freedom. One might object that artistic choice is still limited, because once the beginning of a section is chosen the rest of the texture of the piece is determined. This is only true to a limited extent. Although it is easier to write calm music when the RHS has a sparse texture, it is possible to obtain any kind of musical texture with any structural texture, as the following example from *Danse du feu* proves. Although the RHS is very dense between bars 213 and 216 (see Example 9.36), the music of *Danse du feu* is rather 'ethereal' at this point (see score). In contrast, the corresponding section of *Un souffle de l'air que respirait le passé* (bar 26-28) is rather dense and hectic (see Example 9.37).

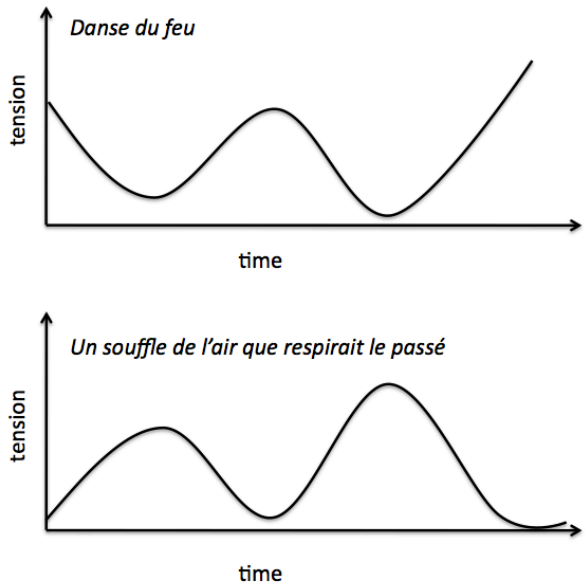


Example 9.36: Bar 213-216 of section D from the RHS of *Danse du feu*.



Example 9.37: Bar 26-28 of *Un souffle de l'air que respirait le passé*.

With this identical RHS for both pieces I composed two pieces that are completely different in sounding structure and evolution of tension, thus showing that the artistic decisions of the composer are more important than the 'objective' and 'cerebral' elements provided by a strict serial technique. Whereas *Danse du feu* starts and ends with a climax (high tension) and has one more climax in the middle, with moments of lower tension in between, the tension curve of *Un souffle de l'air que respirait le passé* has the complete opposite curve, as can be seen in Example 9.38, which shows the tension curves of both pieces. Also the tempo structure and instrumentation of both pieces are completely different. On the level of expression, both pieces are complete opposites: where the general idea of *Danse du feu* is energy and force, of the *hic et nunc*, the idea behind *Un souffle de l'air que respirait le passé* is one of contemplation and melancholy, of things long gone, *à la recherche du temps perdu*.



Example 9.38: Contrasting tension curves of *Danse du feu* (top) and *Un souffle de l'air que respirait le passé* (bottom).

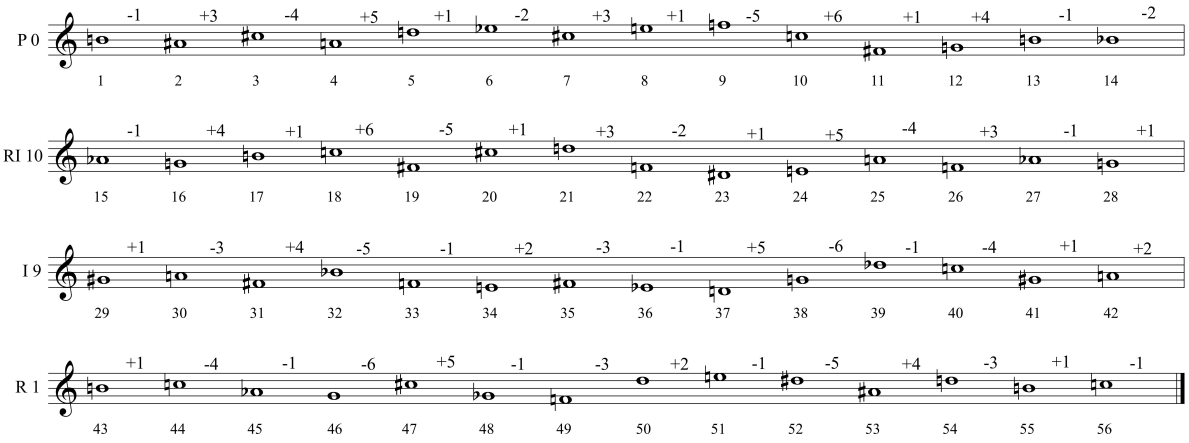
9.6 *Danse de l'eau et de l'air*

9.6.1 Construction of the series

Danse de l'eau et de l'air is the first piece (and only piece within the Elements Project) based on a general CIG-series. All groups of successive notes of any cardinality are CIG's. During the construction of the series, the two Extension Rules described in Section 5.4.4 were observed. The first time this was necessary was after series note 6. As can be seen in the series shown in Example 9.39, the CIG between series notes 4, 5 and 6 (A, D, E flat) ends on ic 1, whereas the CIG is not a permutation of an instance of [3-1]. Therefore the choice of series note 7 (and the CIG-3 between notes 5, 6 and 7) was limited by the first Extension Rule for general CIG-serialism[570]. Note 7 had to be at ic 1 from at least one of the notes of the preceding CIG. This is the case, since note 7 (C sharp) and note 5 (D) are a semitone apart.

The second Extension Rule had to be applied for the first time after the CIG between notes 28, 29 and 30 (G, G sharp, A). The next note (note 31) is at ic 1 distance from note 28, therefore the rule is observed and the series remains a general CIG-series.

[570] See Section 5.4.4.



Example 9.39: The series of *Danse de l'eau et de l'air*.[571]

The series of *Danse de l'eau et de l'air* is a 56-CIG series with a strict internal structure: not only is it invariant (its retrograde is the same as its transposed prime form (P0 = R11), the series can also be divided in four equal subsections each containing 14 notes. Notes 15 to 28 are the same as a transposition of the retrograde inversion of notes 1 to 14 (P0 $_{\text{(notes 1-14)}}$ = RI10 $_{\text{(notes 15-28)}}$), notes 29 to 42 are the same as a transposition of the inversion of notes 1 to 14 (P0 $_{\text{(notes 1-14)}}$ = I9 $_{\text{(notes 29-42)}}$), and notes 43 to 56 are the same as a transposition of the retrograde of notes 1 to 14 (P0 $_{\text{(notes 1-14)}}$ = R1 $_{\text{(notes 43-56)}}$).[572]

9.6.2 Construction of the RHS

9.6.2.1 Pitch distribution in series

The construction of the RHS for *Danse de l'eau et de l'air* started with an assessment of the frequency of occurrence of all different pitch classes in the series. The number of times each pitch class occurs in the series is shown in the table of Example 3.40. As can be seen, the distribution is highly 'balanced'; each pitch class occurs either four or five times.[573]

	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
P0	1	2	1	1	1	1	1	1	0	1	2	2
RI10	1	1	1	1	1	2	1	2	2	1	0	1
I9	1	1	1	1	1	1	2	1	2	2	1	0
R1	2	1	2	1	1	1	1	1	1	0	1	2
total	5	5	5	4	4	5	5	5	5	4	4	5

Example 9.40: Frequency of occurrence of all pitch classes in the four subsections of the series of *Danse de l'eau et de l'air*.

If, together with the original series, a series transposed a major third higher (+4), and one transposed a major third lower (-4 of +8) would be used in the RHS, there would be exactly the same number of rhythmic cells for all pitch classes. This would make the RHS perfectly balanced. We will see that this aim had to be abandoned in order to obtain a suitable distribution of rhythmic cells in the RHS.

[571] The interval class between successive series note is also indicated. As can be noticed, ic 6 is positive (+6) when the ic 5 preceding or following it (i(-n) or i(+n)) is negative and vice versa.
[572] The structure of this series and the series of *Danse du feu* bears similarities to the internal structure of Paul Klee's work *New harmony* (1936) which is used as a cover illustration to the present dissertation (On *New Harmony* by Paul Klee, see also: Allen Shawn, *Arnold Schoenberg's Journey*, Harvard University Press, 2002, pp. 208-9).
[573] Compare this with the 'unbalancedness' of the series of *A l'image du monde…originel* and *A l'image du monde…double*.

9.6.2.2 Number of notes in rhythmic cells

In the RHS of *Danse de l'eau et de l'air*, the number of notes (or note lengths) in each rhythmic cell is proportional to the degree of dissonance of the interval between the series note in question and the neighbouring notes. Rhythmic cells of notes that form larger (most of the time less dissonant)[574] intervals with their direct neighbours contain fewer note lengths than rhythmic cells of notes that form smaller (and generally more dissonant) intervals with their surrounding notes. (Note 4 (A), for instance comes after C sharp and before D, forming rather consonant intervals (ordered ic -4 and +5). Therefore the rhythmic cell of note 4 will consist of less notes than that of, for instance note 1, which is at ic 1 distance from both its neighbours. In order to determine the number of note lengths in the rhythmic cell of a note n, a value X is calculated as the sum of the absolute values (abs) of the interval classes between note n and its direct neighbours (i(-n) and (i(+n)):

X = abs(i(-n)) + abs(+i(+n))

Based on the value of X, the number of note lengths N for the rhythmic cell of note n is than determined according to the distribution shown in the table of Example 9.41.

X	2	3	4	5	6	7	9	11
N	9	8	7	6	5	4	3	2

Example 3.41: Number of note lengths N in rhythmic cells for each value of X.[575]

Note 4 (A), which is at ic distance -4 and +5 from its neighbours has a value for X = 4 + 5 = 9. Therefore N = 3 for note 4, and its rhythmic cell contains three notes.

9.6.2.3 Relative note lengths in rhythmic cells

A first step in the determination of rhythmic cells for all series notes consists of determining the relative note lengths in the cells, that is the note lengths independent of augmentation. The starting relative length in the rhythmic cell of note n is determined as the sum of the absolute values (abs) of the interval classes between note n and its direct neighbours (i(-n) and (i(+n)) modulo 6:

SL(n) = abs(i(-n)) + abs(+i(+n)) (mod6)

For instance:

SL(1) = 1 + 1 = 2
SL(2) = 1+3 = 4
SL(3) = 3 - 4 = 5
SL(4) = 4 + 5 = 3
SL(5) = 5 + 1 = 6
…

For the next relative note lengths in the rhythmic cell of note n, the relative note length on position p ($p \geq 2$) is determined as follows:

[574] Only ic 6 is an exception here.
[575] Values 8 and 10 are absent in the table, because the sum of absolute values of two successive interval classes in the series is never 8 or 10.

Ln(p) = Ln(p-1) + i(+(n+p-2)) (mod6)

Where Ln(p) stands for the relative note lengths in position p within the rhythmic cell of note n, Ln(p-1) for the previous relative note length, and i(+(n+p-2) for the ordered interval class between series note n+p-2 and the next series note. For instance:

Ln(2) = SL(n) + i(+n) (mod6)
Ln(3) = Ln(2) + i(+(n+1)) (mod6)
…

9.6.2.4 Augmentation of rhythmic cells

After determining the relative lengths an augmentation is determined in order to obtain the (absolute) note lengths in all rhythmic cells. As a virtual starting value (not belonging to any series note), AUG(0) is set at 1 (AUG(0) = 1). The general rule for all series notes n in the original (non-transposed) series of *Danse de l'eau et de l'air* is:

AUG(n) = AUG(n-1) +i(-n) + i(+n) (mod6)

For instance:

AUG(1) = 1 + -1 + -1 = -1 = 6
AUG(2) = 6 + -1 + 3 = 8 = 2
AUG(3) = 2 + 3 + -4 = 1
…

In the transposed versions of the series, for every series note: 4 is added to (in transposition +4) or subtracted from (in transposition -4) the augmentation of the original form (modulo 6).

9.6.2.5 Distance between beginnings of rhythmic cells

The distance between the beginnings of the rhythmic cells in the RHS of *Danse de l'eau et de l'air* Depends on the total duration of the piece. Since the duration of the piece will be (ca.) 12 minutes, and the series is used three times consecutively, each part of the RHS based on one occurrence of the series lasts four minutes. At 60 crotchets per minute this means there are 240 crotchets or 960 semiquavers for each consecutive occurrence of the series. This value is approximated when the following formula is applied to calculate the distance of entrance of the rhythmic cell for note n+1 after the beginning of the rhythmic cell for note n:

DIS = [(7-AUG)x(N + abs(+i))/4] + (3 x AUG)

In this formula DIS stands for the distance in semiquaver between the beginnings of the rhythmic cells of notes n and n+1, AUG is the augmentation for note n+1, N its N-value, and +i the ordered interval class between notes n and n+1.

This results in total DIS for the original un-transposed series S_0 = 947, for the transposition +4 S_4 = 938, and for S_8 = 942. These values are just under the pursued 960, which is admissible. However, if only these three series were used in the piece, it would yield distances between rhythmic cells that may lead to undesirable gaps. Therefore extra occurrences of the series needed to be considered. In a first instance I designed a construction where each transposition of the series occurred three times. The core of the construction consisted of the succession of the original series S_0 (with its successive subsections P0, RI10, I9 and R1), the +4 transposition S_4 (P4, RI2, I1 and R5), and the +8 transposition S_8 (P8, RI6, I5 and R9). The core is supplemented as indicated in Example 9.42. As can

be seen, the density of the construction increases towards the middle (where four series are used simultaneously) and decreases back towards the end. This reflects the desired tension curve for the piece and is therefore an example of teleology.

P0	RI10	I9	R1	P4	RI2	I1	R5	P8	RI2	I1	R5
		P4	RI2	I1	R5	P4	RI2	I1	R5		
	P8	RI2	I1	R5	P8	RI2	I1	R5			
			P0	RI10	I9	R1	P0	RI10	I9	R1	

Example 9.42: Combination of series forms with each the transposition of the series occurring three times (the original series S_0 is indicated with a grid background, S_4 without background and S_8 in grey).

The structure proved to be too dense, however. I chose to restrict it in order to obtain a sparser texture in the piece. This resulted in the structure represented in Example 9.43. In this structure, the original core of three transpositions is complemented with two inversions of the series, the inversion on the original series (consisting of I0, R4, P3, RI1, indicated with horizontal lines in Example 9.43) and the inversion transposed nine semitones higher (which is identical to S_0 starting at note 29: I9, R1, P0, RI10, indicated with vertical lines in Example 9.43).

P0	RI10	I9	R1	P4	RI2	I1	R5	P8	RI2	I1	R5
			I9	R1	P0	RI10					
					I0	R4	P3	RI1			

Example 9.43: Succession of the the series in the RHS of *Danse de l'eau et de l'air*. The core is complemented with two versions of the inversion of the original series (indicated with horizontal and vertical lines).

The second time the inversion occurs (bottom row in Example 9.43) it is un-transposed (starting on B), the first time (middle row in Example 9.43) it is transposed a minor third lower (+9; starting on G sharp). This way, the choice of transpositions for the inversions results in a highly 'balanced' distribution of pitch classes in the combined series (although not all series notes occur the same number of times in the inversions), as the list in Example 9.44 shows.

	C	C#	D	D#	E	F	F#	G	G#	A	A#	B
Inv_0	4	4	5	5	5	5	4	4	5	5	5	5
Inv_9	5	5	5	4	4	5	5	5	5	4	4	5
total	9	9	10	9	9	10	9	9	10	9	9	10

Example 9.44: Distribution of pitch classes in the inversion of the series used in *Danse de l'eau et de l'air*. Each pitch class occurs 9 or 10 times, which makes the construction highly balanced.

In the RHS of *Danse de l'eau et de l'air*, each new version of the series starts at the corresponding section of the series in the core of the structure. The first section of the +9 transposition of the inversion (I9) for instance starts simultaneously with the rhythmic cell of note 43 of S_0 (the first note of section R1 in S_0 (B natural)). All other rules for the construction of rhythmic cells and their distribution in time remain unchanged.

9.7 *A l'image du monde…originel & double*

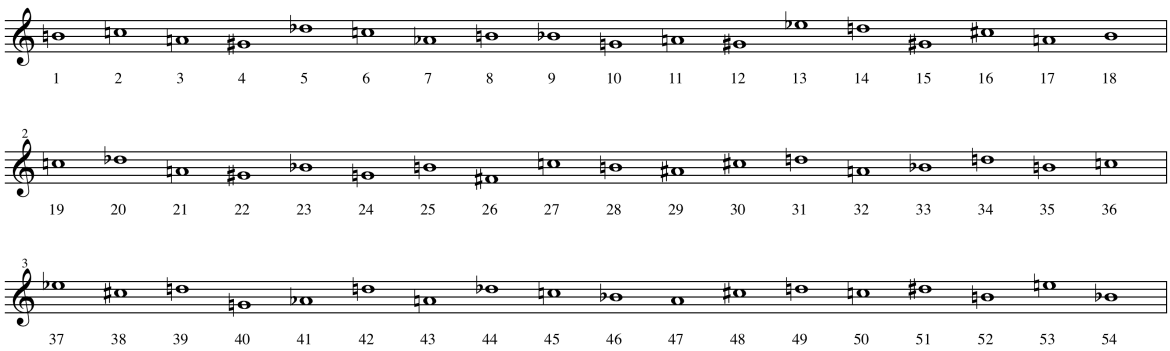
9.7.1 Introduction

As was discussed above, in order to show that the impact of the RHS of a piece is limited, and that the creative role of the composer is prevalent even in strictly 'constructed' music such as that composed with the technique of CIG-serialism, I wrote two completely differently sounding pieces (*Danse du feu* and *Un souffle de l'air que respirait le passé*) with the same RHS. This proved that one and the same RHS may lead to at least two different results; it doesn't prove, however, that the composer has total control over the result. It may still be the case that not all imaginable outcomes are possible with a specific RHS. Of course, this is exact to a certain extent; it is even the point that I wanted to make: CIG-serialism necessarily yields highly atonal and dissonant music. That certainly limits the possibilities of the technique, but that was exactly the aim of the present research. It does not mean, however, that a composer has only limited choices as to the sounding result. A specific RHS may yield any sound result that a composer may have in mind. To prove this claim, I composed two pieces that are essentially 'identical' starting from different series and RHS's: *A l'image du monde…originel* and *A l'image du monde…double*, the two piano pieces that accompany *Danse de l'eau et de l'air*. The second of the piano pieces (*A l'image du monde…double*) was also composed in the context of the POINT project led by dr. Gerhard Nierhaus at the KunstUni Graz (Austria).[576]

9.7.2 *A l'image du monde…originel*

9.7.2.1 Construction of the series

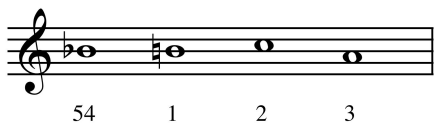
With *A l'image du monde…originel* (2012) I wanted to explore the extreme registers of the piano. Since the cluster A-C occurs at both ends of the instrument's register, I constructed a 3/4-CIG series that started and ended on the pitch classes belonging to the cluster A-C, as can be seen in the series in Example 9.45 a and b.



Example 9.45 a: The series of *A l'image du monde…originel*.[577]

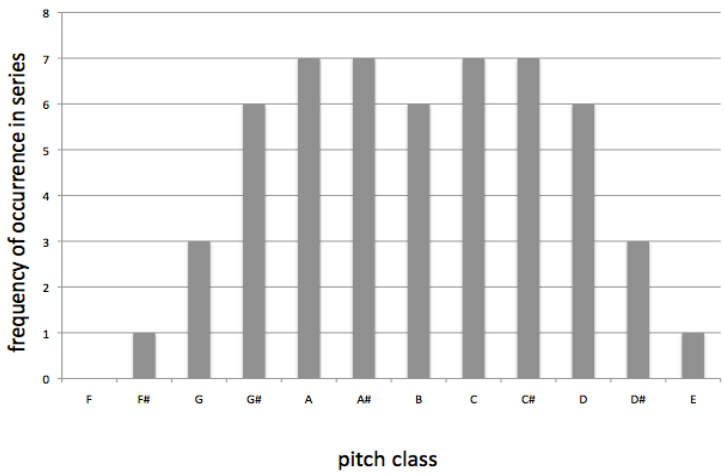
[576] For an extended description of my contribution to this project, see: Bart Vanhecke, *Straightening the Tower of Pisa*. In Gerhard Nierhaus (ed.), *Patterns of Intuition, Musical Creativity in the Light of Algorithmic Composition.* Berlin & Heidelberg: Springer Verlag, forthcoming. Part of the present text is taken from my contribution to the book.

[577] Note that there is a mutation in *A l'image du monde…originel*: series note 18 'should be' B flat instead of B natural. This mutation originates from a mistake, but as a result, the right hand plays B and C at the end of bar 6 (instead of possibly A and B flat), which is in a more extreme register than it would have been without the mutation. I see it therefore as an improvement. This anomaly was mentioned to me by Daniel Mayer in the context of the POINT-project.



Example 9.45 b: Notes 54-1 in the series of *A l'image du monde…originel*, containing all the pitch classes of cluster A-C.

A second feature of the series of *A l'image du monde…originel* is that its second half (notes 28-54) is the inversion of its first half (notes 1-27). In addition, during the process of exploration of the series, I noticed that the series of *A l'image du monde…originel* had an interesting but unpremeditated feature: it is unbalanced, meaning that the frequency of occurrence of the different pitch classes is not homogeneously distributed. As can be seen in Example 9.46, showing the pitch class distribution of the series of *A l'image du monde…originel*, the pitch classes belonging to cluster A-C occur more frequently (on average) than the other pitch classes. Although this feature was not intended, it serves the aim of exploring the extreme registers of the piano perfectly. A second remarkable feature is the absence of pitch class F in the series of *A l'image du monde…originel*.



Example 9.46: Pitch class distribution in the series of *A l'image du monde…originel.*

In *A l'image du monde…originel*, each form of the series is used once in the following order of appearance: prime form, retrograde inversion, inversion and finally retrograde.

9.7.2.2 Determination of the number of notes in each rhythmic cell in the RHS

The RHS for *A l'image du monde…originel* is based on the interval class information contained in the series. First the number of note lengths in each rhythmic cell was determined. N is defined as the number of note lengths in the rhythmic cell for series note n; it is calculated with the following formula:

N = integer (S/4 + W/2)

where

S = abs(-i) + abs(+i).[578]

[578] 'abs' stands for 'absolute value'.

-i is the interval class between note n and the preceding note in the series; +i is the interval class between n and the note following n in the series. 'Integer' means that the result of 'S/4 + W/2' is rounded off upwards or downwards to the nearest integer.

W= abs(+i + -i) + frequency of occurrence of the pitch class n in the series.

N varies between 3 and 8. For the first note in the series (B natural, occurring six times in the whole series), for instance, S = 2, W = 2 + 6 = 8, and therefore N = integer(2/4 + 8/2) = 4.

9.7.2.3 Determination of the relative note lengths in each rhythmic cell

The information to determine (relative) note lengths is also taken from the interval class values for the series. In this process I distinguish between prime form (P) and inversion (I) on the one hand and retrograde (R) and retrograde inversion (RI) on the other:

For P and I, the series of unordered interval class numbers is run through in forward direction (from left to right):

1 3 1 5 1 4 3 1 3 2 1 5 1 6 5 4 1 2 1 4 1 2 3 4 5 6 1

For R and RI, it is run through backwards:

Each series notes is attributed N consecutive numbers from the list, starting with the first number for note 1.

Note 1 (N = 4): 1 3 1 5

The next series notes continues the list after the last number of the previous series note:

Note 2 (N = 5): 1 4 3 1 3
Note 3 (N = 6): 2 1 5 1 6 5
…

A complete list is provided in Examples 9.47 a and b.

9.7.2.4 Determination of augmentations

In *A l'image du monde…originel*, the augmentations for rhythmic cells (AUG) are determined as the sum of the values for relative note lengths of the rhythmic cell (modulo 6).

ex: for note 1 AUG = 1+3+1+5 (mod 6) = 4

This yields a lot of high numbers (esp. 6) in the beginning of the prime form of the series. Since I intended a hectic climax at the beginning and end of the piece, the long values are unsuitable. Therefore, as a case of teleological mutations, the numbers are turned around for P but kept unchanged for I (simile for RI en R respectively).

For R and I: AUG = sum of note lengths of rhythmic cell modulo 6
For P and RI: AUG = 6 – (sum of note lengths of rhythmic cell modulo 6) + 1

e.g.: AUG for note 1 = 3 in version P.

The result is given in lists in Examples 9.47 a and b.

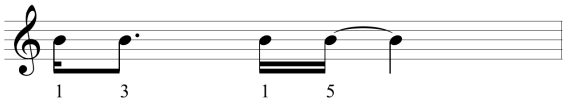
Note	N	relative note lengths	augmentation P	augmentation I
1	4	1 3 1 5	3	4
2	5	1 4 3 1 3	1	6
3	6	2 1 5 1 6 5	5	2
4	6	4 1 2 1 4 1	6	1
5	7	2 3 4 5 6 1 1	3	4
6	7	3 1 5 1 4 3 1	1	6
7	5	3 2 1 5 1	1	6
8	5	6 5 4 1 2	1	6
9	6	1 4 1 2 3 4	4	3
10	3	5 6 1	1	6
11	4	1 3 1 5	3	4
12	7	1 4 3 1 3 2 1	4	3
13	6	5 1 6 5 4 1	3	4
14	8	2 1 4 1 2 3 4 5	3	4
15	6	6 1 1 3 1 5	2	5
16	6	1 4 3 1 3 2	5	4
17	6	1 5 1 6 5 4	3	4
18	5	1 2 1 4 1	4	3
19	5	2 3 4 5 6	5	2
20	6	1 1 3 1 5 1	1	6
21	7	4 3 1 3 2 1 5	6	1
22	4	1 6 5 4	3	4
23	5	1 2 1 4 1	4	3
24	3	2 3 4	4	3
25	5	5 6 1 1 3	3	4
26	3	1 5 1	6	1
27	7	4 3 1 3 2 1 5	6	1
28	4	1 6 5 4	3	4
29	5	1 2 1 4 1	4	3
30	6	2 3 4 5 6 1	4	3
31	6	1 3 1 5 1 4	4	3
32	7	3 1 3 2 1 5 1	3	4
33	7	6 5 4 1 2 1 4	2	5
34	5	1 2 3 4 5	4	3
35	5	6 1 1 3 1	1	6
36	6	5 1 4 3 1 3	2	5
37	3	2 1 5	5	2
38	4	1 6 5 4	3	4
39	7	1 2 1 4 1 2 3	5	2
40	6	4 5 6 1 1 3	5	2
41	8	1 5 4 3 1 3 2 1	5	2
42	6	5 1 6 5 4 1	3	4
43	6	2 1 4 1 2 3	6	1
44	6	4 5 6 1 1 3	5	2
45	5	1 5 1 4 3	5	2
46	5	1 3 2 1 5	1	6
47	6	1 6 5 4 1 2	6	1
48	7	1 4 1 2 3 4 5	5	2
49	4	6 1 1 3	2	5
50	5	1 5 1 4 3	5	2
51	3	1 3 2	1	6
52	5	1 5 1 6 5	1	6
53	3	4 1 2	6	1
54	7	1 4 1 2 3 4 5	5	2

Example 9.47 a: Rhythmic cells for P and I of *A l'image du monde…originel*.

Note	N	relative note lengths	augmentation R	augmentation RI
1	4	1 6 5 4	4	3
2	5	3 2 1 4 1	5	2
3	6	2 1 4 5 6 1	1	6
4	6	5 1 2 3 1 3	3	4
5	7	4 1 5 1 3 1 1	4	3
6	7	6 5 4 3 2 1 4	1	6
7	5	1 2 1 4 5	1	6
8	5	6 1 5 1 2	3	4
9	6	3 1 3 4 1 5	5	2
10	3	1 3 1	5	2
11	4	1 6 5 4	4	3
12	7	3 2 1 4 1 2 1	2	5
13	6	4 5 6 1 5 1	4	3
14	8	2 3 1 3 4 1 5 1	2	5
15	6	3 1 1 6 5 4	2	5
16	6	3 2 1 4 1 2	1	6
17	6	1 4 5 6 1 5	4	3
18	5	1 2 3 1 3	4	3
19	5	4 1 5 1 3	2	5
20	6	1 1 6 5 4 3	2	5
21	7	2 1 4 1 2 1 4	3	4
22	4	5 6 1 5	5	2
23	5	1 2 3 1 3	4	3
24	3	4 1 5	4	3
25	5	1 3 1 1 6	6	1
26	3	5 4 3	6	1
27	7	2 1 4 1 2 1 4	3	4
28	4	5 6 1 5	5	2
29	5	1 2 3 1 3	4	3
30	6	4 1 5 1 3 1	3	4
31	6	1 6 5 4 3 2	3	4
32	7	1 4 1 2 1 4 5	6	1
33	7	6 1 5 1 2 3 1	1	6
34	5	3 4 1 5 1	2	5
35	5	3 1 1 6 5	4	3
36	6	4 3 2 1 4 1	3	4
37	3	2 1 4	1	6
38	4	5 6 1 5	5	2
39	7	1 2 3 1 3 4 1	3	4
40	6	5 1 3 1 1 6	5	2
41	8	5 4 3 2 1 4 1 2	4	3
42	6	1 4 5 6 1 5	4	3
43	6	1 2 3 1 3 4	2	5
44	6	1 5 1 3 1 1	6	1
45	5	6 5 4 3 2	2	5
46	5	1 4 1 2 1	3	4
47	6	4 5 6 1 5 1	4	3
48	7	2 3 1 3 4 1 5	1	6
49	4	1 3 1 1	6	1
50	5	6 5 4 3 2	1	6
51	3	1 4 1	6	1
52	5	2 1 4 5 6	6	1
53	3	1 5 1	1	6
54	7	2 3 1 3 4 1 5	1	6

Example 9.47 b: Rhythmic cells for R and RI of *A l'image du monde…originel*.

Knowing the rhythmic cell of note 1 has four durations (1, 3, 1 and 5) in augmentation 3 (1= semiquaver, 2 = quaver, etc.) the rhythmic cell for note 1 in the prime form (P) of the series is shown in Example 9.48:



Example 9.48: Rhythmic cell for note 1
in the prime form (P) of *A l'image du monde…originel*.

9.7.2.5 Determination of the distance between beginning of rhythmic cells

The whole piece is meant to last ca. 5 minutes. There are four occurrences of the series (once in every form P, I, R and RI, (see above)). The beginning and ending are faster and more energetic than the (calmer) middle section. The following temporal structure was put forward:

Tempo:	♩ = 80	♩ = 40	♩ = 80	
portion of time:	25%	50%	25%	
number of beats:	100	100	100	total: 300 beats (5 minutes)
series form:	P	RI & I	R	

The sum of all values for S (see Section 9.7.2.2) is 304. This is close to the 4 times 75, the number of beats that can be accorded to each of the four series forms in order to obtain a piece of ±300 beats. The distance between beginnings of rhythmic cells of note x and next occurring note (DISx) is therefore determined as the number of semiquavers (1/4 beats) equal to S for note x. For triplets (AUG 2 and 4), the distance is rounded off to the semiquaver closest to the calculated distance.

As another case of a teleological mutation, for R and I the series of values of S is turned around (7 11 9 7 5 …) because the series contains lower values in the beginning and higher values at the end and I want the piece to begin and end with denser rhythm.

For P and R: DISx = Sx semiquavers
For I and RI: DISx = S(55-x) semiquavers.

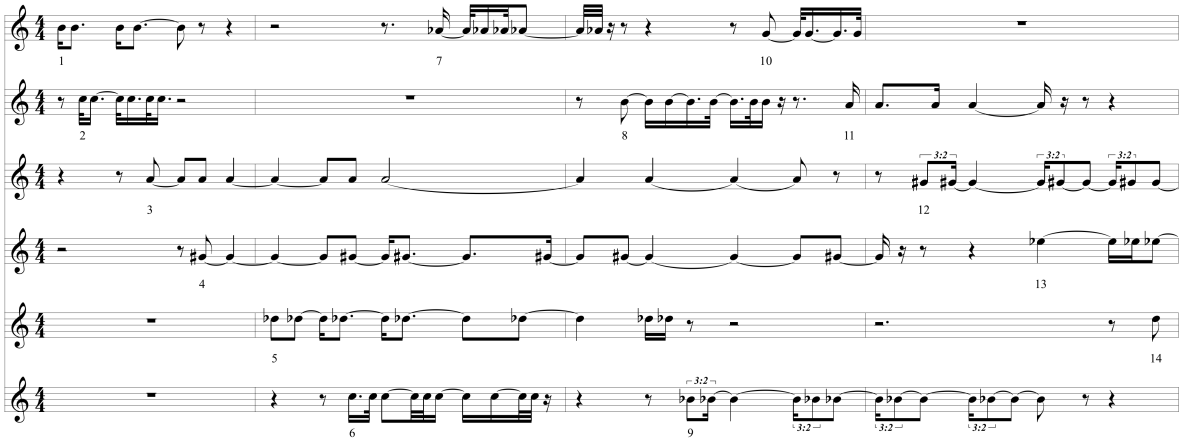
An overview of the distribution of DIS over the four forms of the series is shown in Example 9.49.

form	notes	DIS
P	1→54	1→54
RI	54→1	1→54
I	1→54	54→1
R	54→1	54→1

Example 9.49: Distribution of DIS over the four forms
of the series of *A l'image du monde…originel.*

..

The distance between the start of rhythmic cells 1 and 2 in the RHS: DIS(1) = 2. This means the rhythmic cell of note 2 starts 2 semiquavers later than that of note 1. The first four bars of the RHS resulting from this procedure is shown in Example 9.50:



Example 9.50: Bar 1-4 of the RHS of *A l'image du monde…originel*.

9.7.3 *A l'image du monde…double*

A l'image du monde…double was composed in the context of the Patters of Intuition (POINT) project led by dr. Gerhard Nierhaus at the KunstUni Graz. In this project, a research team explores "the creative act of composing by means of algorithmic composition"[579] The creative processes of the composers involved were investigated and simulated by algorithmic processes. For me, this was an exquisite opportunity to compose a 'double' for *A l'image du monde…originel*, a second piano piece that is essentially identical to its 'original': *A l'image du monde…double*. This piece was meant to have the same essential characteristics as *A l'image du monde…originel* although the two pieces would be based on different series and RHS's.

In order to obtain a double that is essentially identical to the original, the relevant criteria and characteristics of the two pieces had to be the same, not just in the surface structure (the score), but also in the sub-structural levels (series and RHS).

The construction of series for *A l'image du monde…originel* was done according to three main criteria:

1. The series is a 54-note 3/4-CIG series.
2. The beginning and end contain all pitch classes of the cluster A-C.
3. The second half of the series is the inversion of the first half.

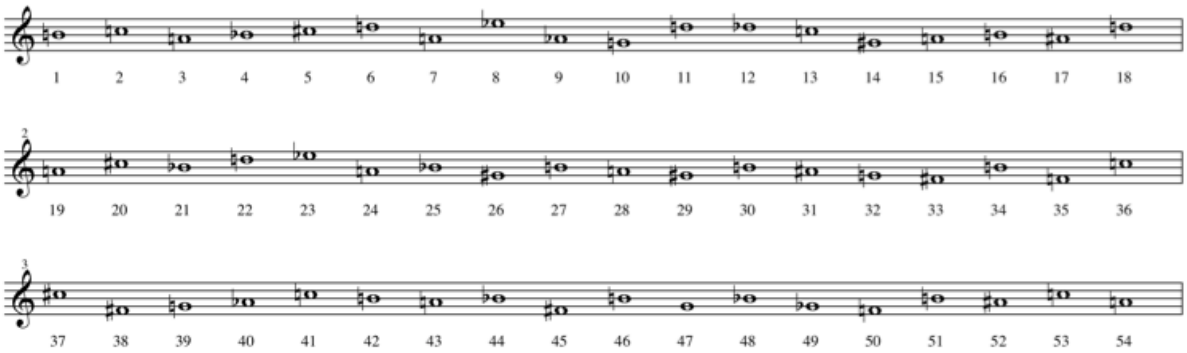
These criteria were imposed by me; they are the result of deliberate choice, of self-imposed restriction, not of restrictions inherent to a compositional technique. In addition to these three criteria, the series appeared to have two more important characteristics:

1. The series of *A l'image du monde…originel* is unbalanced, containing the pitch classes belonging to the cluster A-C more frequently than the others.

[579] See abstract of: Gerhard Nierhaus (ed.), *Patterns of Intuition, Musical Creativity in the Light of Algorithmic Composition.* Berlin & Heidelberg: Springer Verlag, 2014.

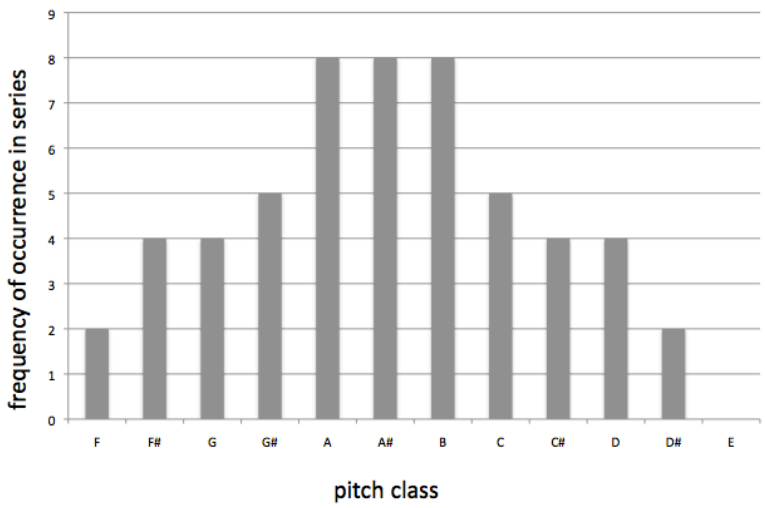
2. Pitch class F is absent from the series of *A l'image du monde…originel*. Together with E, F is the pitch class furthest away from the centre of the cluster A-C.

These five criteria and characteristics are relevant for the composition of *A l'image du monde…originel*. In combination with the construction rules for the RHS, they are the sub-structural backbone of the piece. These very same characteristics were used in the construction of the series and RHS of *A l'image du monde…double*. The POINT-researchers constructed algorithms for the construction of a series that possesses all of them. In a modelling process, a computer calculated dozens of such series, of which I chose the one that seemed the most appropriate to me. This series is shown in Example 9.51.



Example 9.51: The computer generated series of *A l'image du monde…double.*

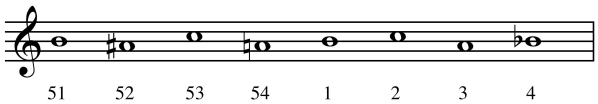
The graph representing the pitch class distribution in Example 9.52 reveals that the series of *A l'image du monde…double* is more outspokenly unbalanced than the series of *A l'image du monde…originel*. Indeed, relatively the pitch classes of cluster A-C occur relatively more frequently than the others in the series of *A l'image du monde…double* than in the series of the 'original' (compare Example 9.52 with Example 9.46).



Example 9.52: Pitch class distribution in the series of *A l'image du monde…double*.[580]

[580] Note that in the series of *A l'image du monde…double*, pitch class E is absent instead of pitch class F.

The A-C clusters at the 'beginning' and 'end' of the series of *A l'image du monde…double* (shown in Example 9.53) is much more extended than the cluster in the series of *A l'image du monde…originel* (see Example 9.45 b). It is also much more symmetrically spread: four notes at both 'beginning' and 'end' of the series.[581]



Example 9.53: A-C cluster between notes 51 and 4
in the series of *A l'image du monde…double*.

The next step in the composition process was to construct a RHS for *A l'image du monde…double*. This was done on the basis of exactly the same formulas as were used in *A l'image du monde…originel*. This step could therefore be considered entirely 'mine'. Since the series of *A l'image du monde…double* is different from that of *A l'image du monde…originel*, it has a different interval class content, and therefore starts from different data, resulting in a different RHS.

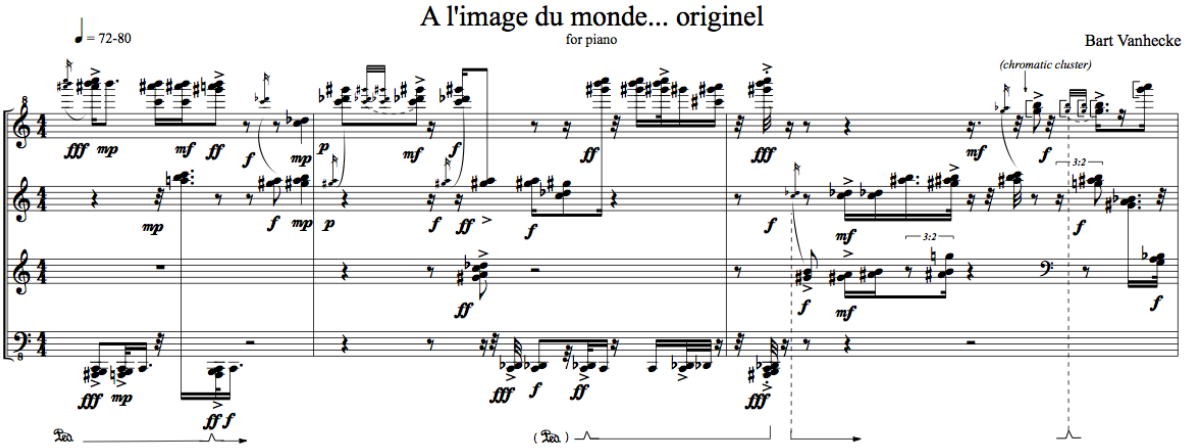
The contribution of the POINT project stopped at this point. The transformation of the RHS into a surface structure (the score) was left up to me. As was mentioned above, I tried to compose *A l'image du monde…double* in such a manner that *A l'image du monde…originel* and *A l'image du monde…double* were *essentially* identical. Since they start from a different RHS, they could not be *strictly* identical, but I tried to make them so similar that it should become impossible to say which piece is the original and which one the double. *A l'image du monde…double* should not be perceived as a variation of *A l'image du monde…originel* any more than the other way around. In order to achieve this, I used not only the series and RHS of *A l'image du monde…double* for the composition of the 'double', but at all times I had the score of *A l'image du monde…originel* in front of me. I tried to 'copy' *A l'image du monde…originel* with the material of *A l'image du monde…double*. This way, the pieces became as similar to each other as, for instance, the different *Marilyn Monroe portraits* by Andy Warhol, or Arnold Böcklin's five versions of the painting *Die Toteninsel* (*The Isle of the Dead*). Examples 9.54 a and b show the opening bars of both twin pieces.

Because of its more outspoken characteristics, the series of *A l'image du monde…double* may be called 'more perfect' than the series of *A l'image du monde…originel*. The former piece meets the criteria better. This raises important questions about authorship and artistic perfection: if the series of *A l'image du monde…double* is 'more perfect' than my own series, is *A l'image du monde…double* then a more perfect piece than *A l'image du monde…originel*? Can I legitimately call the computer-generated series mine?

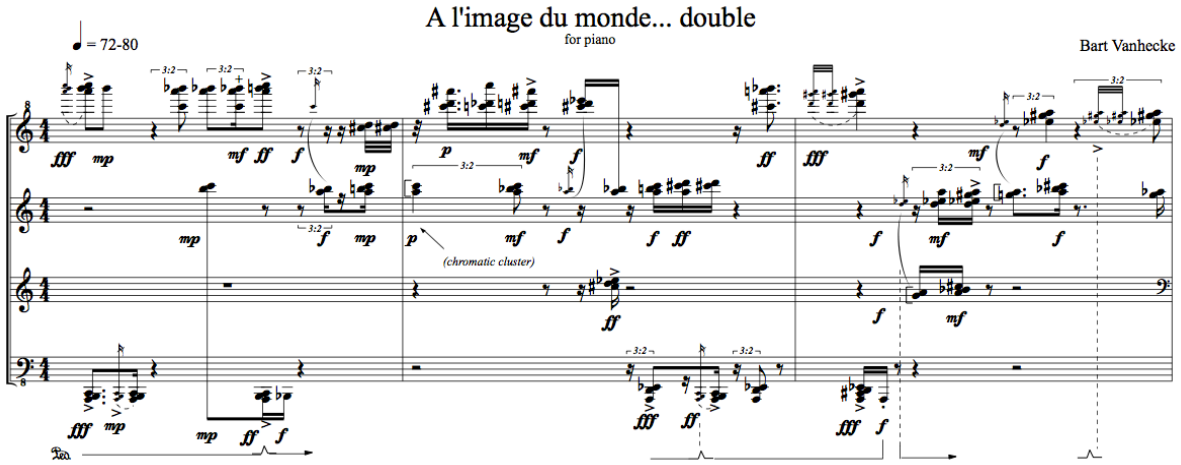
In order to answer these questions it is important to distinguish between voluntary and involuntary elements in the process of composition, between controlled and automatic cognitive processes on the series construction level.[582] The construction of the series for *A l'image du monde…originel* involved controlled processes, such as the deliberate choice for a CIG-3/4 series, the central A-C cluster and the inversive symmetry of the series, but also processes that escaped my control completely, such as the fact that the series is unbalanced and does not contain pitch class F, or to a large extent, such as the limited possibilities of successive CIG's. Comparing *A l'image du monde…originel* with *A l'image du monde…double* showed that all the controlled *essential* elements of *A l'image du monde…originel* are also controlled by me according to my criteria for the construction of the series of *A l'image du*

[581] The symmetrical distribution of the large A-C cluster in the series is actually the element that determined my choice for this series among the dozens of series generated by the computer.
[582] The processes of construction of the RHS and of transformation of the RHS into the score were identical on the level of control in both cases, as was discussed above.

monde…double. The essential elements that were not controlled by me in the series of *A l'image du monde…double* were the result of automatic computer processes, but those elements escaped my control anyway. Therefore I concluded that I was entitled to be called the sole composer of *A l'image du monde…double*. The computer is no more the co-author of *A l'image du monde…double* than the pencil I used in the composition of *A l'image du monde…originel* is the co-author of the 'original' piece. The composition of *A l'image du monde…double* did not escape my control any more than the composition of *A l'image du monde…originel* .



(a)



(b)

Example 9.54 a & b: Bars 1-3 from (a) *A l'image du monde…originel* and
(b) *A l'image du monde…double,* illustrating the striking resemblance between both pieces.

The computer was not more than a tool in the composition of *A l'image du monde…double*, just like the pencil I use was only a tool in the composition of *A l'image du monde…originel*. Of course, my pencil *does* help me in finding the optimal solution. Without a writing tool of some sort, I would not be able to structure my pieces the way I do. It helps me visualise my composition. What the computer did for me is help me construct a series. I normally use a kind of domino card system (with all CIG-3's) as a tool to construct a series. As far as I was concerned, he computer played just that role: an electronic domino card system.

The series of *A l'image du monde…double* can be called 'more perfect' than the series of *A l'image du monde…originel*, in the sense that it meets the construction criteria better. This does not entail however that the 'double' is aesthetically more perfect than the 'original' version. Perfection is not a criterion for aesthetic value. It is often the case that voluntary or involuntary imperfections add aesthetic value to the artworks. The *campanile* of Pisa would probably not have been as attractive and fascinating, or as famous, if it would not have the imperfection that makes it lean over dangerously.[583] The greater perfection of the series of *A l'image du monde…double* doesn't make the 'double' aesthetically any more valuable than *A l'image du monde…originel* in my eyes. I cannot even subjectively call one series 'more perfect' than another.

By composing two contrasting pieces with the same substructure and two essentially identical pieces with different calculated material I showed the relative impact of that material on the end result. The strictly calculated starting conditions only have a very limited influence on the end result that is still completely determined by the aesthetic judgment of the composer. Indeed, a piece is determined by the material it is composed with, just like human beings are determined by their purely biochemical genetic material. Still, the same genetic material can result in completely different personalities. External influences, education and experience play a major role in formation of a personality. Similarly, the composer's artistic personality plays the most important role in all artistic creation, even the most rigorously strict one. This crucial part of the composition process, the artistic transformation of the RHS into the surface structure (the score), is only superficially discussed in the present thesis, because it is the least measurable, the least assessable, and the most intuitive part of the process, the part that mostly escapes verbal description. It is also the most personal part of my artistic practice, and therefore the least relevant on a more general level.

9.7.4 T-analysis of *A l'image du monde…originel*

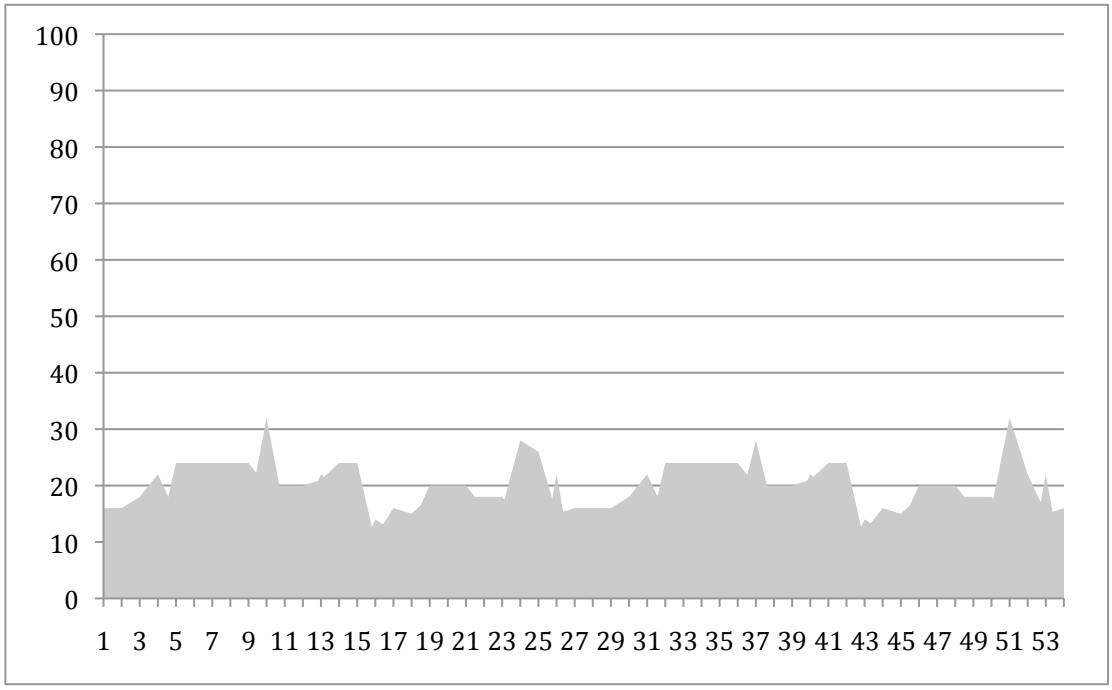
The present research showed that general CIG-serialism leads to highly atonal and dissonant music in a systematic way. Indeed, the degree of tonality of a general CIG- series can never exceed a T-value of 32 (when the series notes combine to form an instance of [6-z3i]). In the series of *A l'image du monde…originel* this happens twice. This can be seen in Example 9.55, listing all possible T-values in the series. A graph of the highest T-values for each note (indicated in Grey in Example 9.55) is shown in Example 9.56).

For each series note, the T-values of all ordered pc-sets contained in the series that end on the series note in question is given in the list of Example 9.55. Note 1 (B natural), for instance forms an instance of [2-1] with the 'previous' note (note 54, B flat); there's an instance of [3-5] between notes 53 to 1 as well as between notes 52 and 1 (E, B flat, B), an instance of [4-8] between notes 51 and 1 (D sharp, E, B flat, B) and so on. The corresponding T-values for note 1 can be read in the first row of Example 9.55. The highest T-value in this row is 16 for 6-z4 between notes 49 and 1 (D, C, D sharp, E, B flat, B). The highest T-value in the entire series is reached at series notes 10 and 51, both instances of [6-z3i]. This doesn't mean that the degree of tonality of *A l'image du monde…originel* will be at its highest level at the occurrence of notes 10 and 51; it only guarantees it will not be higher than 32.

[583] These remarks are (in slightly different form) also part of my article in the POINT-book (see: Bart Vanhecke, *Straightening the Tower of Pisa*. In Gerhard Nierhaus (ed.), *Patterns of Intuition, Musical Creativity in the Light of Algorithmic Composition.* Berlin & Heidelberg: Springer Verlag, forthcoming). The title of the article refers to the idea of 'perfection' in the case of a 'straight Leaning Tower'.

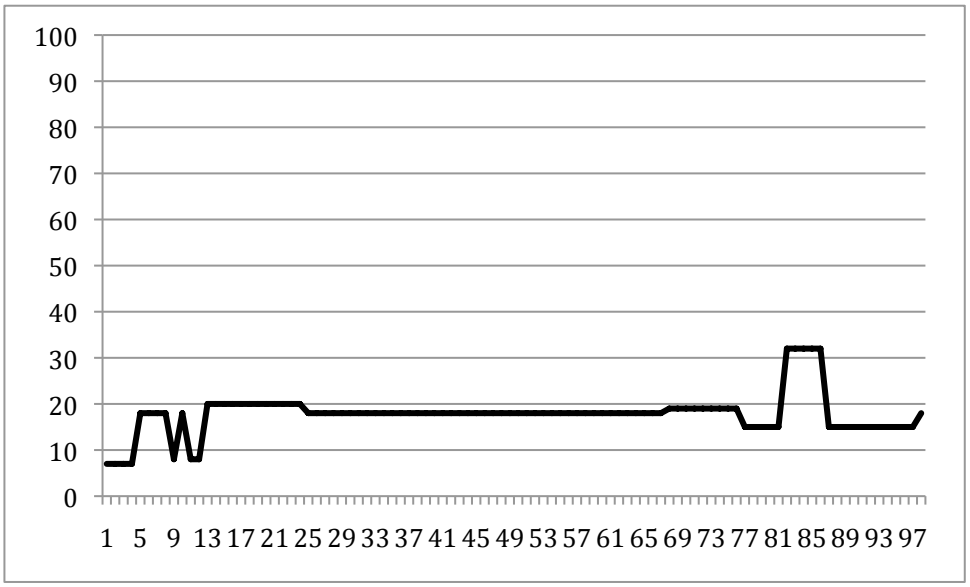
Note nr																				
1	2-1	9	3-5	12	4-8	11	5-6	8	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
2	2-1	9	3-1	7	4-5	8	5-6	8	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
3	2-3	14	3-2	18	4-1	8	5-5	13	6-5	18	7-5	13	8-1	3	9-1	2	10-1	2	11-1	2
4	2-1	9	3-3	15	4-3	20	5-1	6	6-z37	22	7-6	9	8-5	10	9-1	2	10-1	2	11-1	2
5	2-5	14	3-4	16	4-7	16	5-3i	18	6-1	15	7-3	19	8-4	24	9-1	2	10-1	2	11-1	2
6	2-1	9	3-4i	16	4-7	16	5-3i	18	6-1	15	7-3	19	8-4	24	9-1	2	10-1	2	11-1	2
7	2-4	12	3-4i	16	4-7	16	5-3i	18	6-1	15	7-3	19	8-4	24	9-1	2	10-1	2	11-1	2
8	2-3	14	3-3	15	4-4i	13	5-3i	18	6-1	15	7-3	19	8-4	24	9-1	2	10-1	2	11-1	2
9	2-1	9	3-2i	18	4-2i	12	5-2i	19	6-1	15	7-3	19	8-4	24	9-1	2	10-1	2	11-1	2
10	2-3	14	3-3i	15	4-3	20	5-3i	18	6-z3i	32	7-1	4	8-3	20	9-3	22	10-1	2	11-1	2
11	2-2	13	3-2i	18	4-2i	12	5-1	6	6-1	15	7-1	4	8-3	20	9-3	22	10-1	2	11-1	2
12	2-1	9	3-1	7	4-1	8	5-1	6	6-1	15	7-1	4	8-3	20	9-3	22	10-1	2	11-1	2
13	2-5	14	3-5i	11	4-5i	8	5-5i	13	6-z37	22	7-3	19	8-2	21	9-3	22	10-1	2	11-1	2
14	2-1	9	3-5	12	4-9	6	5-7i	5	6-z38	19	7-6	9	8-4	24	9-1	2	10-1	2	11-1	2
15	2-6	7	3-5	12	4-9	6	5-7i	5	6-z38	19	7-6	9	8-4	24	9-1	2	10-1	2	11-1	2
16	2-5	14	3-5i	11	4-6	10	5-7i	5	6-7	4	7-7	4	8-5	10	9-1	2	10-1	2	11-1	2
17	2-4	12	3-4	16	4-8	11	5-7i	5	6-7	4	7-7	4	8-5	10	9-1	2	10-1	2	11-1	2
18	2-1	9	3-3	15	4-4	13	5-6	8	6-z6	12	7-7	4	8-5	10	9-1	2	10-1	2	11-1	2
19	2-2	13	3-2	18	4-3	20	5-3	18	6-z4	16	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
20	2-1	9	3-2i	18	4-3	20	5-3	18	6-z4	16	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
21	2-4	12	3-3i	15	4-3	20	5-3	18	6-z4	16	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
22	2-1	9	3-4	16	4-7	16	5-3	18	6-z4	16	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
23	2-2	13	3-1	7	4-4	13	5-3	18	6-z4	16	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
24	2-3	14	3-2	18	4-1	8	5-4	14	6-z3	28	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
25	2-4	12	3-3i	15	4-3	20	5-1	6	6-2	26	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
26	2-5	14	3-4	16	4-7	16	5-3	18	6-1	15	7-2	22	8-1	3	9-1	2	10-1	2	11-1	2
27	2-6	7	3-5i	11	4-8	11	5-6i	10	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
28	2-1	9	3-5i	11	4-8	11	5-6i	10	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
29	2-1	9	3-1	7	4-5i	8	5-6i	10	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
30	2-3	14	3-2	18	4-1	8	5-5i	13	6-5i	18	7-5i	13	8-1	3	9-1	2	10-1	2	11-1	2
31	2-1	9	3-3i	15	4-3	20	5-1	6	6-z37	22	7-6i	16	8-5i	10	9-1	2	10-1	2	11-1	2
32	2-5	14	3-4i	16	4-7	16	5-3	18	6-1	15	7-3i	20	8-4i	24	9-1	2	10-1	2	11-1	2
33	2-1	9	3-4	16	4-7	16	5-3	18	6-1	15	7-3i	20	8-4i	24	9-1	2	10-1	2	11-1	2
34	2-4	12	3-4	16	4-7	16	5-3	18	6-1	15	7-3i	20	8-4i	24	9-1	2	10-1	2	11-1	2
35	2-3	14	3-3	15	4-4	13	5-3	18	6-1	15	7-3i	20	8-4i	24	9-1	2	10-1	2	11-1	2
36	2-1	9	3-2	18	4-2	12	5-2	19	6-1	15	7-3i	20	8-4i	24	9-1	2	10-1	2	11-1	2
37	2-3	14	3-3	15	4-3	20	5-3	18	6-z3	28	7-1	4	8-3	20	9-3i	22	10-1	2	11-1	2
38	2-2	13	3-2	18	4-2	12	5-1	6	6-1	15	7-1	4	8-3	20	9-3i	22	10-1	2	11-1	2
39	2-1	9	3-1	7	4-1	8	5-1	6	6-1	15	7-1	4	8-3	20	9-3i	22	10-1	2	11-1	2
40	2-5	14	3-5i	11	4-5	8	5-5	13	6-z37	22	7-3i	20	8-2i	21	9-3i	22	10-1	2	11-1	2
41	2-1	9	3-5i	11	4-9	6	5-7	5	6-z38	19	7-6i	16	8-4i	24	9-1	2	10-1	2	11-1	2
42	2-6	7	3-5	12	4-9	6	5-7	5	6-z38	19	7-6i	16	8-4i	24	9-1	2	10-1	2	11-1	2
43	2-5	14	3-5	12	4-6	10	5-7	5	6-7	4	7-7i	4	8-5i	10	9-1	2	10-1	2	11-1	2
44	2-4	12	3-4i	16	4-8	11	5-7	5	6-7	4	7-7i	4	8-5i	10	9-1	2	10-1	2	11-1	2
45	2-1	9	3-3i	15	4-4i	13	5-6i	10	6-z6	12	7-7i	4	8-5i	10	9-1	2	10-1	2	11-1	2
46	2-2	13	3-2i	18	4-3	20	5-3i	18	6-z4	16	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
47	2-1	9	3-2	18	4-3	20	5-3i	18	6-z4	16	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
48	2-4	12	3-3	15	4-3	20	5-3i	18	6-z4	16	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
49	2-1	9	3-4i	16	4-7	16	5-3i	18	6-z4	16	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
50	2-2	13	3-1	7	4-4i	13	5-3i	18	6-z4	16	7-5	13	8-6	17	9-1	2	10-1	2	11-1	2
51	2-3	14	3-2i	18	4-1	8	5-4i	14	6-z3i	32	7-5i	13	8-6	17	9-1	2	10-1	2	11-1	2
52	2-4	12	3-3	15	4-3	20	5-1	6	6-2i	22	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2
53	2-5	14	3-4i	16	4-7	16	5-3i	18	6-1	15	7-2i	22	8-1	3	9-1	2	10-1	2	11-1	2
54	2-6	7	3-5	12	4-8	11	5-6	8	6-z4	16	7-1	4	8-1	3	9-1	2	10-1	2	11-1	2

Example 9.55: List of T-values in the analysis of the series of *A l'image du monde...original*. The highest value for each series note is indicated in grey.



Example 9.56: Maximum T-values in series of *A l'image du monde…originel*
(unit of the horizontal axis = demi-semiquaver).

The highest T-value of 32 is actually reached in *A l'image du monde…originel.* This happens on the third beat of bar 3, where series notes 8 and 10 form instances of 6-z3i as a result of the chord rule. This level is only sustained for a length of 5 demi-semiquavers however. It comes at a short stop and is immediately followed by a loud and accentuated chromatic cluster (see the score in Example 9.54 a), lowering the degree of tonality back to T(6-1) = 15. The short moment of raised degree of tonality can be seen around unit 85 in the graph in Example 9.57 representing the T-curve for the first three bars of *A l'image du monde…originel*.



Example 9.57: T-curve for *A l'image du monde…originel* bar 1-3
(unit of the horizontal axis = demi-semiquaver).

The list with T-values and PC-values in Appendix 1 might be used during the process of composition of the score to choose sound combinations in such a way that always the lowest possible degree of tonality is obtained and occurrences of instances of [6-3i] are avoided. I did not use this procedure in the composition of *A l'image du monde…originel*, because it would have limited my choice as a composer too much and it may have disturbed the homogeneity of the sound result. Also, the relatively high T-value for [6-z3i] may just be the result of an imperfection in the T-formula.

Conclusion and further directions

1. Conclusion

The search for an answer to the central research question of the present dissertation—whether the technique of CIG-serialism yields highly atonal and dissonant music and whether this result can be enhanced—started from Reginald Smith Brindle's assumption about the characteristics of what he calls 'atonal series'. The development of the initial technique of CIG-serialism (which was renamed 'CIG-3-serialism' as a result of the present research) was based on this assumption. In order to find an answer to the central research question, the concepts of tonality/atonality and consonance/dissonance were examined first.

Chapter 3 of the dissertation describes how, by analysing the most generally accepted definitions of the concept of tonality, and by distilling the common defining factor of diatonicity from those definitions, the concept of tonality can be redefined in such a manner that quantification of degrees of tonality becomes possible. This resulted in the construction of the T-formula, used to determine the degree of tonality of any pitch class set belonging to any set class.

A technique of tonality analysis (T-analysis) was then developed on the basis of the T-formula. The technique is applicable in the analysis of music of all periods, idioms and styles, as long as they are based on the use of the pitch classes of the chromatic scale (set class [12-1])[584] or its subsets, and is thus applicable to a broad field of musical investigation. This increases the general relevance of what would otherwise have been a research of personal interest only. Examples of analyses were given, ranging for highly diatonic music (music with a high degree of tonality), over chromatic music and extended tonality, to highly atonal music. The T-formula allows for an investigation into how tonality evolved in music history, and supplies a possible answer to the problem of extended tonality and the question whether highly chromatic music of certain 'problematic' composers (such as Richard Wagner, Max Reger, or the young Arnold Schoenberg, to name just some) should be called tonal or atonal. With classical definitions and analysis methods, it is sometimes hard to determine whether music belonging to the idioms of extended tonality is either tonal or atonal; with the T-formula and T-analysis, this music shows to have a lowered degree of tonality, without however totally moving to the side of 'atonal' music (which has a degree of tonality that rarely or never reaches the maximum value of 100).

Application of the T-formula in T-analysis revealed also that the original technique of 54-note CIG-3-serialism does yield highly atonal music, which confirms the first part of the basic conjecture of my thesis. Indeed, CIG-3 serialism is based on the occurrence of CIG-3's (exclusively in the series and predominantly in the resulting music) and these have the lowest degree of tonality within the set class group of cardinality 3.

In Chapter 4, the concept of consonance was assessed in a similar way. Different types of consonance were first discussed: harmonic and melodic consonance, musical and sensory consonance. Consonance indices for all interval classes were then determined based on the 'objective' criterion of 'simplest frequency ratios of sound combinations'. The concept of just noticeable difference (JND) for pitch difference discrimination was used in combination with Clarence Barlow's idea of 'bending into place' to justify the use of simple frequency ratios for intervals in Equal Temperament (ET). The consonance indices thus determined were used as a basis for the construction of a formula that allows

[584] [12-1] (and the numbers for pitch class sets with 2 and 11 elements appearing in the list in the appendix alike) is no regular Forte number; it was introduced in the text by me for convenience, as was mentioned in the list of usages.

for quantification of consonance. This resulted in the PC-formula for the determination of the degree of prime-consonance (the degree of consonance of pitch class sets in prime form) of all pitch class sets belonging to all possible set classes that use the pitch classes of the chromatic scale in ET.

The PC-formula confirmed the initial assumption that CIG-3-serialism yields highly dissonant music, since the structural units of CIG-3-serialism (CIG-3's) showed to be the most dissonant within their cardinality group.

A technique of prime consonance analysis (PC-analysis) was developed on the basis of the PC-formula. The implementation of this technique in the analysis of musical scores showed how, together with the degree of tonality, degrees of consonance evolved over the centuries towards ever-lower values. Although tonality and consonance are distinct aspects of music, the formulas for the quantification of degrees of tonality and prime consonance revealed a correlation between tonality and consonance in Western common-practice tonality.

It is important to note that the T-formula and PC-formula developed in the Part 1 of the preceding text are not the only possible ones. The T-formula offers one of many possible ways of quantifying tonality. A different approach might lead to another formula that would, however, probably yield similar results. The same applies to the PC-formula, where different consonance indices (as they are developed by other researchers) may result in different PC-values. Any formula in accordance with the reality of tonal perception and with the spirit of the widest possible range of definitions could serve the purpose. What is important in the two formulas is not the absolute but the relative values of the degrees of tonality they determine they yield.

It is also important to realize that the tonality and consonance formulas tell us *how* those phenomena work and *what* they are within my aesthetic universe, not *why*. This is also characteristic for scientific laws, as Richard Feyman notices in the context of Newton's laws:

> Newton was originally asked about his theory—'But it doesn't mean anything—it doesn't tell us anything'. He said, 'It tells you *how* [a planet] moves. That should be enough. I have told you how it moves, not why.'[585]

The second goal of the research—the second part of the central research question—was to assess whether the technique of CIG-serialism could be adapted in such a manner that the degree of tonality and prime consonance of the resulting music could be kept even lower than with the original technique in a systematic way. This assessment was done in Chapter 5. As a starting point in this process, the concept of chromatic interval group (CIG) was extended to set classes of orders higher than 3 (containing more than 3 pitch classes). Within each cardinality group of set classes, the subgroup to which the CIG's belong appeared to contain the set classes with the lowest degree of tonality and prime consonance. This entails that series consisting exclusively of CIG's of any cardinality will systematically yield music with the lowest degrees of tonality and prime consonance. This observation culminated in the following adaptations of the technique of CIG-serialism: extension rules for CIG's of any order were determined and a mathematical proof was given that the extension on CIG-3's and CIG-4's automatically lead to new CIG's except in a restricted number of cases, and that the extension of CIG-5's always results in CIG's.[586] Two restriction rules for the construction of series that consist exclusively of CIG's were distilled from the extension rules. By adding the two restriction rules to the original construction rules, the initial technique of CIG-3-serialism was transformed into a technique based on the use of series in which successive series notes (pitch classes) of any number constitute a CIG. This restricted technique was called general CIG-serialism. The less restricted initial technique was renamed CIG-3-serialism, and the intermediate stage of CIG-3/4-serialism (that only applies the

[585] Richard Feynman, *The character of physical law*, Penguin books, 1965 (1992), p. 37.
[586] Although this mathematical proof is the crux of the present dissertation, it has been included as an appendix (Appendix 3) in order not to impair the legibility of the text.

First Extension Rule) was discussed and implemented in composition during the second phase of the research. In addition, two 3-note CIG-2's were added as structural units, resulting in 56-CIG-series.

The technique of general CIG-serialism is probably not the only technique that results in highly atonal and dissonant music. Other techniques—even non-serial ones—may be developed, but this was not the purpose of the present dissertation, which constitutes artistic research (research on personal artistic practice) and not comprehensive musicological or general theoretical research. I am fully aware that, even so, Part 1 is highly theoretical and that it contains a lot of mathematical and other 'objective' considerations. This may prompt questions about the link with the 'subjective' artistic output of the present research. There may seem to be a gap between theory and practice in my activity as a composer. Part 2 of the dissertation addresses the aesthetic considerations of my practice as a composer and provides a necessary answer to these questions. It demonstrates that the gap is only apparent and it explains the link between theory and practice. For this purpose, the concept of the artist's aesthetic universe—the set consisting of all the artist's aesthetic knowledge—was introduced. Chapter 6 develops the ideas belonging to semiotics and epistemology necessary for the introduction of the concept of aesthetic universe. Chapter 7 describes how the technique of CIG-serialism is the tool, the space probe, with which my personal, highly idiosyncratic aesthetic universe is explored and expressed. It also explains that the T-formula and PC-formula represent the endophysical laws of my aesthetic universe.

Finally, Part 3 brings the ideas of the two preceding parts together: the compositions that constitute the artistic output of the research are the result of the exploration and expression of combined technical and aesthetic ideas. It is therefore a synthesis of the musico-theoretical and aesthetic ideas developed in the preceding parts, resulting in the Elements Project. This project consists of seven pieces for different instrumental combinations with at its core three orchestral pieces representing the four Empedoclian elements of my personal aesthetic universe. The project reflects three phases in the research: the initial situation of CIG-3 serialism, the phase implementing intermediate outcomes of the research (a.o. CIG-3/4-serialism), and finally the phase where general CIG-serialism is put to the test.

It will be noticed that there is no directly noticeable audible difference worth mentioning in the sounding result between the pieces of the different phases of the research (or even compared to my previous works). This was one of the aims of my research from the outset: to preserve the idiosyncrasies of my personal style whilst developing the serial technique of CIG-serialism. Does this make my research highly irrelevant—if one can't hear any difference, why go through all the toil? I don't think so. It is not because the differences are inaudible that they are necessarily irrelevant. Or to say it with the words of Nicholas Cook:

> Most contemporary theorists feel uncomfortable about ascribing significance to inaudible relationships in music; we tend to assume that there should be some meaningful relationship between analysis and auditory experience. There is no obligatory reason why this should be so.[587]

Having said that, it is not completely true that my musical idiom and the compositions it produces have not developed as a result of the present research. First of all, the assessment of my technique has led to an increase of structural coherence of my music; the enhanced internal structure of the series I use (invariant halves, for instance) has become standard as a result of the research. Also, the simultaneous use of different series forms or different transpositions of the series has become more controlled in order to avoid 'tonal' combinations. And finally, the change in the sounding result of my compositions is largely situated in what is *not* heard. Whereas passages with increased degrees of tonality and consonance occasionally occur in CIG-3 serialism, such passages become impossible or at least rare and easily avoidable with the more rigorous technique of general CIG-serialism.

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[587] Nicholas Cook, *Music Theory and 'Good Comparison': A Viennese Perspective*, Journal of Music Theory, Vol. 33, N°1, 1989, p. 117.

Even if the audible result of my artistic practice hasn't markedly changed, my aesthetic universe and my awareness of the place of my aesthetic universe within the broader aesthetic cultural context it belongs to—or deviates from—together with the effectiveness of the tool I use to express it—CIG-serialism—have evolved considerably as a result of my research of the last six years. In that sense, I am convinced that my research has made me evolve as an artist and composer.

2. Further directions

My artistic practice has always been interwoven with my activities as an artistic researcher. My aesthetic ideas are in constant evolution as a result of an ongoing venture into the undiscovered territories of my personal aesthetic universe; vice versa, my interest in artistic inquiry is triggered by my activities as a composer. The modest contribution that I want to make to the development of musical culture is an unending task, and the path this dual quest will follow is not clearly delineated. Although I have clear ideas about the pieces I am planning to compose in the (relatively) near future, I cannot predict—not even imaging—how my music will evolve in the further future. Indeed, if I could, I would write that music today and it would no longer belong to the future. Since my artistic research is tightly related to my artistic practice, its future direction is also unclear to a certain extent.

Therefore, I am aware of the fact that the research described in the present dissertation is far from complete. Some of the specific ideas that have been developed in it are open to revision, and elements that need further investigation can in some cases easily be pointed out. The formulas for the determination of degrees of tonality and prime-consonance, for instance, are probably not in their final form as they are at present. They require further fine-tuning. This is not necessarily a shortcoming of the present research. Artistic research is in this respect not different from (pure) scientific research, where formulas describing physical laws, for instance, are under constant testing and are adjusted and 'perfected'—made more generally valid and more in accordance with the facts of reality—according to newly acquired knowledge if need be. The relatively high T-value for set class [6-z3i], for instance, is an element in the present research outcome that needs to be assessed further. But, as was mentioned, even when the criteria used in the formulas would be adjusted, the results would probably stay essentially similar; the absolute values might change, but the relative values would probably not be meaningfully different. The proof of the extension rules for CIG's would in any case not be affected, since it depends neither on the concepts of tonality and dissonant, notron the T- and PC-formulas, but is purely mathematical. Even when different concepts and formulas would be used, the proof will stay unaffected and it will probably not change the conclusion that CIG's (ordered chromatic pitch class sets) are cases of the most dissonant set classes within every cardinality group either.

Digitalisation of T-analysis and PC-analysis is another possible further step in the development of the present research. The T-analyses and PC-analyses performed in Chapters 3 and 4 were all done 'by hand'. This manual analysis is a time consuming and tedious activity. It is certainly possible to develop adequate software to facilitate both methods of analysis. This would make it easier to analyse high numbers of 'problematic' scores. It will show exceptions in music history as well as possible flaws or inaccuracies in the formulas and the analysis methods. The implementation of the methods of T- and PC-analysis after digitalisation may thus challenge the analysis methods themselves. This may lead to further adaptations or fine-tuning of the T-formula and PC-formula as they are now, and possibly also to new insights into the nature of tonality and consonance, which may in turn lead to modifications in the concepts developed in the present dissertation. With the necessary software—similar to the software developed in the POINT-project mentioned in Chapter 9—it must also be possible to calculate the CIG-series with the absolute lowest degrees of tonality and prime consonance. In this respect, I want to remark that the development of software exceeds the borders of artistic research and that it can only be performed with the help of computer analysts. Other topics of further research belong to the specialised 'cerebral universes' of mathematicians, music theorists, or musicologists, because they are not directly related to the exploration and development of any aesthetic universe, and therefore lie beyond the domain of artistic research.

Appendix 1
Degree of T-values and PC-values for all set classes

(set classes of CIG's are indicated in grey).

Set class	$T_{(c-n)}$	$PC_{(c-n)}$
2-1 (ic 1)	9	26
2-2 (ic 2)	13	66
2-3 (ic 3)	14	86
2-4 (ic 4)	12	93
2-5 (ic 5)	14	100
2-6 (ic 6)	7	57

3-1	7	20
3-2	18	37
3-2i	18	37
3-3	15	45
3-3i	15	45
3-4	16	49
3-4i	16	49
3-5	12	37
3-5i	11	37
3-6	19	65
3-7	22	78
3-7i	22	78
3-8	15	60
3-8i	17	60
3-9	22	84
3-10	20	66
3-11	22	94
3-11i	20	94
3-12	14	95

4-1	8	15
4-2	12	26
4-2i	12	26
4-3	20	29
4-4	13	31
4-4i	13	31
4-5	8	26
4-5i	8	26
4-6	10	27
4-7	16	36
4-8	11	31
4-9	6	25
4-10	30	44
4-11	26	45
4-11i	26	45
4-12	21	40
4-12i	25	40
4-13	27	41
4-13i	24	41
4-14	30	54
4-14i	26	54
4-z15	21	42
4-z15i	16	42
4-16	22	45
4-16i	22	45
4-17	21	58
4-18	18	47
4-18i	23	47
4-19	24	60
4-19i	20	60

4-20	23	62
4-21	22	54
4-22	30	78
4-22i	33	78
4-23	36	81
4-24	22	62
4-25	17	51
4-26	31	87
4-27	33	69
4-27i	28	69
4-28	25	59
4-z29	18	42
4-z29i	22	42

5-1	6	12
5-2	19	20
5-2i	19	20
5-3	18	22
5-3i	18	22
5-4	14	19
5-4i	14	19
5-5	13	20
5-5i	13	20
5-6	8	22
5-6i	10	22
5-7	5	20
5-7i	5	20
5-8	20	25
5-9	19	27
5-9i	17	27
5-10	43	28
5-10i	32	28
5-11	26	36
5-11i	26	36
5-z12	36	30
5-13	17	30
5-13i	17	30
5-14	24	32
5-14i	22	32
5-15	13	27
5-16	22	32
5-16i	33	32
5-z17	41	40
5-z18	32	34
5-z18i	34	34
5-19	20	28
5-19i	20	28
5-20	25	36
5-20i	34	36
5-21	32	45
5-21i	23	45
5-22	28	38
5-23	58	50
5-23i	50	50
5-24	38	43
5-24i	38	43
5-25	52	45

5-25i	42	45
5-26	44	45
5-26i	42	45
5-27	41	57
5-27i	50	57
5-28	33	38
5-28i	30	38
5-29	51	48
5-29i	52	48
5-30	42	48
5-30i	32	48
5-31	46	41
5-31i	37	41
5-32	40	51
5-32i	29	51
5-33	30	48
5-34	52	64
5-35	59	80
5-z36	22	30
5-z36i	24	30
5-z37	24	40
5-z38	22	34
5-z38i	22	34

6-1	15	10
6-2	26	14
6-2i	22	14
6-z3	28	16
6-z3i	32	16
6-z4	16	16
6-5	18	16
6-5i	18	16
6-z6	12	17
6-7	4	14
6-8	48	26
6-9	41	23
6-9i	37	23
6-z10	36	24
6-z10i	39	24
6-z11	46	25
6-z11i	46	25
6-z12	32	22
6-z12i	32	22
6-z13	40	23
6-14	43	34
6-14i	47	34
6-15	35	28
6-15i	35	28
6-16	35	29
6-16i	31	29
6-z17	27	24
6-z17i	30	24
6-18	37	25
6-18i	33	25
6-z19	41	32
6-z19i	58	32
6-20	38	43

6-21	38	29
6-21i	37	29
6-22	37	30
6-22i	34	30
6-z23	65	31
6-z24	87	39
6-z24i	71	39
6-z25	65	40
6-z25i	84	40
6-z26	59	41
6-27	75	34
6-27i	49	34
6-z28	67	35
6-z29	73	36
6-30	46	30
6-30i	43	30
6-31	48	44
6-31i	70	44
6-32	87	68
6-33	87	56
6-33i	87	56
6-34	63	46
6-34i	67	46
6-35	43	48
6-z36	21	16
6-z36i	24	16
6-z37	22	16
6-z38	19	17
6-z39	40	24
6-z39i	37	24
6-z40	40	25
6-z40i	40	25
6-z41	34	22
6-z41i	33	22
6-z42	36	23
6-z43	29	24
6-z43i	33	24
6-z44	32	32
6-z44i	33	32
6-z45	49	31
6-z46	49	39
6-z46i	48	39
6-z47	52	40
6-z47i	56	40
6-z48	50	41
6-z49	45	35
6-z50	50	36

7-1	4	8
7-2	22	13
7-2i	22	13
7-3	19	15
7-3i	20	15
7-4	15	12
7-4i	15	12
7-5	13	13
7-5i	13	13
7-6	9	15
7-6i	16	15
7-7	4	13

7-7i	4	13
7-8	30	17
7-9	35	18
7-9i	28	18
7-10	38	19
7-10i	30	19
7-11	49	24
7-11i	50	24
7-z12	30	20
7-13	20	20
7-13i	21	20
7-14	41	21
7-14i	35	21
7-15	14	18
7-16	34	21
7-16i	42	21
7-z17	30	27
7-z18	27	23
7-z18i	34	23
7-19	31	19
7-19i	32	19
7-20	24	24
7-20i	26	24
7-21	37	31
7-21i	34	31
7-22	31	26
7-23	63	34
7-23i	61	34
7-24	49	29
7-24i	50	29
7-25	66	31
7-25i	59	31
7-26	41	31
7-26i	47	31
7-27	64	39
7-27i	66	39
7-28	43	26
7-28i	44	26
7-29	53	32
7-29i	60	32
7-30	49	33
7-30i	40	33
7-31	60	27
7-31i	47	27
7-32	100	35
7-32i	63	35
7-33	42	33
7-34	100	44
7-35	100	56
7-z36	39	20
7-z36i	46	20
7-z37	43	27
7-z38	41	23
7-z38i	41	23

8-1	3	6
8-2	21	11
8-2i	21	11
8-3	20	12
8-4	24	13
8-4i	24	13
8-5	10	11
8-5i	10	11
8-6	17	11

8-7	17	15
8-8	9	13
8-9	8	10
8-10	44	19
8-11	38	20
8-11i	38	20
8-12	33	17
8-12i	30	17
8-13	39	18
8-13i	37	18
8-14	38	24
8-14i	41	24
8-z15	44	19
8-z15i	31	19
8-16	28	20
8-16i	28	20
8-17	52	26
8-18	36	21
8-18i	48	21
8-19	33	27
8-19i	30	27
8-20	40	28
8-21	47	24
8-22	68	37
8-22i	70	37
8-23	69	38
8-24	34	28
8-25	31	23
8-26	76	41
8-27	69	32
8-27i	58	32
8-28	54	27
8-z29	30	19

9-1	2	4
9-2	20	9
9-2i	20	9
9-3	22	12
9-3i	22	12
9-4	16	13
9-4i	16	13
9-5	16	9
9-5i	12	9
9-6	36	18
9-7	46	22
9-7i	47	22
9-8	25	16
9-8i	29	16
9-9	41	25
9-10	39	18
9-11	46	28
9-11i	43	28
9-12	24	28

10-1	2	9
10-2	23	13
10-3	29	15
10-4	20	16
10-5	26	8
10-6	15	9

11-1	2	2

12-1	2	1

Appendix 2
Interval class vectors of all set classes

(CIG's are indicated in grey)

Set Class	Interval vector					
2-1 (ic 1)	1	0	0	0	0	0
2-2 (ic 2)	0	1	0	0	0	0
2-3 (ic 3)	0	0	1	0	0	0
2-4 (ic 4)	0	0	0	1	0	0
2-5 (ic 5)	0	0	0	0	1	0
2-6 (ic 6)	0	0	0	0	0	1
3-1	2	1	0	0	0	0
3-2	1	1	1	0	0	0
3-2i	1	1	1	0	0	0
3-3	1	0	1	1	0	0
3-3i	1	0	1	1	0	0
3-4	1	0	0	1	1	0
3-4i	1	0	0	1	1	0
3-5	1	0	0	0	1	1
3-5i	1	0	0	0	1	1
3-6	0	2	0	1	0	0
3-7	0	1	1	0	1	0
3-7i	0	1	1	0	1	0
3-8	0	1	0	1	0	1
3-8i	0	1	0	1	0	1
3-9	0	1	0	0	2	0
3-10	0	0	2	0	0	1
3-11	0	0	1	1	1	0
3-11i	0	0	1	1	1	0
3-12	0	0	0	3	0	0
4-1	3	2	1	0	0	0
4-2	2	2	1	1	0	0
4-2i	2	2	1	1	0	0
4-3	2	1	2	1	0	0
4-4	2	1	1	1	1	0
4-4i	2	1	1	1	1	0
4-5	2	1	0	1	1	1
4-5i	2	1	0	1	1	1
4-6	2	1	0	0	2	1
4-7	2	0	1	2	1	0
4-8	2	0	0	1	2	1
4-9	2	0	0	0	2	2
4-10	1	2	2	0	1	0
4-11	1	2	1	1	1	0
4-11i	1	2	1	1	1	0
4-12	1	1	2	1	0	1
4-12i	1	1	2	1	0	1
4-13	1	1	2	0	1	1
4-13i	1	1	2	0	1	1
4-14	1	1	1	1	2	0
4-14i	1	1	1	1	2	0
4-z15	1	1	1	1	1	1
4-z15i	1	1	1	1	1	1
4-16	1	1	0	1	2	1
4-16i	1	1	0	1	2	1
4-17	1	0	2	2	1	0
4-18	1	0	2	1	1	1
4-18i	1	0	2	1	1	1
4-19	1	0	1	3	1	0
4-19i	1	0	1	3	1	0
4-20	1	0	1	2	2	0
4-21	0	3	0	2	0	1
4-22	0	2	1	1	2	0
4-22i	0	2	1	1	2	0
4-23	0	2	1	0	3	0
4-24	0	2	0	3	0	1
4-25	0	2	0	2	0	2
4-26	0	1	2	1	2	0
4-27	0	1	2	1	1	1
4-27i	0	1	2	1	1	1
4-28	0	0	4	0	0	2
4-z29	1	1	1	1	1	1
4-z29i	1	1	1	1	1	1
5-1	4	3	2	1	0	0
5-2	3	3	2	1	1	0
5-2i	3	3	2	1	1	0
5-3	3	2	2	2	1	0
5-3i	3	2	2	2	1	0
5-4	3	2	2	1	1	1
5-4i	3	2	2	1	1	1
5-5	3	2	1	1	2	1
5-5i	3	2	1	1	2	1
5-6	3	1	1	2	2	1
5-6i	3	1	1	2	2	1
5-7	3	1	0	1	3	2
5-7i	3	1	0	1	3	2
5-8	2	3	2	2	0	1
5-9	2	3	1	2	1	1
5-9i	2	3	1	2	1	1
5-10	2	2	3	1	1	1
5-10i	2	2	3	1	1	1
5-11	2	2	2	2	2	0
5-11i	2	2	2	2	2	0
5-z12	2	2	2	1	2	1
5-13	2	2	1	3	1	1
5-13i	2	2	1	3	1	1
5-14	2	2	1	1	3	1
5-14i	2	2	1	1	3	1
5-15	2	2	0	2	2	2
5-16	2	1	3	2	1	1
5-16i	2	1	3	2	1	1
5-z17	2	1	2	3	2	0
5-z18	2	1	2	2	2	1
5-z18i	2	1	2	2	2	1
5-19	2	1	2	1	2	2
5-19i	2	1	2	1	2	2
5-20	2	1	1	2	3	1
5-20i	2	1	1	2	3	1
5-21	2	0	2	4	2	0
5-21i	2	0	2	4	2	0
5-22	2	0	2	3	2	1
5-23	1	3	2	1	3	0
5-23i	1	3	2	1	3	0
5-24	1	3	1	2	2	1
5-24i	1	3	1	2	2	1
5-25	1	2	3	1	2	1
5-25i	1	2	3	1	2	1

5-26	1	2	2	3	1	1
5-26i	1	2	2	3	1	1
5-27	1	2	2	2	3	0
5-27i	1	2	2	2	3	0
5-28	1	2	2	2	1	2
5-28i	1	2	2	2	1	2
5-29	1	2	2	1	3	1
5-29i	1	2	2	1	3	1
5-30	1	2	1	3	2	1
5-30i	1	2	1	3	2	1
5-31	1	1	4	1	1	2
5-31i	1	1	4	1	1	2
5-32	1	1	3	2	2	1
5-32i	1	1	3	2	2	1
5-33	0	4	0	4	0	2
5-34	0	3	2	2	2	1
5-35	0	3	2	1	4	0
5-z36	2	2	2	1	2	1
5-z36i	2	2	2	1	2	1
5-z37	2	1	2	3	2	0
5-z38	2	1	2	2	2	1
5-z38i	2	1	2	2	2	1

6-1	5	4	3	2	1	0
6-2	4	4	3	2	1	1
6-2i	4	4	3	2	1	1
6-z3	4	3	3	2	2	1
6-z3i	4	3	3	2	2	1
6-z4	4	3	2	3	2	1
6-5	4	2	2	2	3	2
6-5i	4	2	2	2	3	2
6-z6	4	2	1	2	4	2
6-7	4	2	0	2	4	3
6-8	3	4	3	2	3	0
6-9	3	4	2	2	3	1
6-9i	3	4	2	2	3	1
6-z10	3	3	3	3	2	1
6-z10i	3	3	3	3	2	1
6-z11	3	3	3	2	3	1
6-z11i	3	3	3	2	3	1
6-z12	3	3	2	2	3	2
6-z12i	3	3	2	2	3	2
6-z13	3	2	4	2	2	2
6-14	3	2	3	4	3	0
6-14i	3	2	3	4	3	0
6-15	3	2	3	4	2	1
6-15i	3	2	3	4	2	1
6-16	3	2	2	4	3	1
6-16i	3	2	2	4	3	1
6-z17	3	2	2	3	3	2
6-z17i	3	2	2	3	3	2
6-18	3	2	2	2	4	2
6-18i	3	2	2	2	4	2
6-z19	3	1	3	4	3	1
6-z19i	3	1	3	4	3	1
6-20	3	0	3	6	3	0
6-21	2	4	2	4	1	2
6-21i	2	4	2	4	1	2
6-22	2	4	1	4	2	2
6-22i	2	4	1	4	2	2
6-z23	2	3	4	2	2	2
6-z24	2	3	3	3	3	1
6-z24i	2	3	3	3	3	1

6-z25	2	3	3	2	4	1
6-z25i	2	3	3	2	4	1
6-z26	2	3	2	3	4	1
6-27	2	2	5	2	2	2
6-27i	2	2	5	2	2	2
6-z28	2	2	4	3	2	2
6-z29	2	2	4	2	3	2
6-30	2	2	4	2	2	3
6-30i	2	2	4	2	2	3
6-31	2	2	3	4	3	1
6-31i	2	2	3	4	3	1
6-32	1	4	3	2	5	0
6-33	1	4	3	2	4	1
6-33i	1	4	3	2	4	1
6-34	1	4	2	4	2	2
6-34i	1	4	2	4	2	2
6-35	0	6	0	6	0	3
6-z36	4	3	3	2	2	1
6-z36i	4	3	3	2	2	1
6-z37	4	3	2	3	2	1
6-z38	4	2	1	2	4	2
6-z39	3	3	3	3	2	1
6-z39i	3	3	3	3	2	1
6-z40	3	3	3	2	3	1
6-z40i	3	3	3	2	3	1
6-z41	3	3	2	2	3	2
6-z41i	3	3	2	2	3	2
6-z42	3	2	4	2	2	2
6-z43	3	2	2	3	3	2
6-z43i	3	2	2	3	3	2
6-z44	3	1	3	4	3	1
6-z44i	3	1	3	4	3	1
6-z45	2	3	4	2	2	2
6-z46	2	3	3	3	3	1
6-z46i	2	3	3	3	3	1
6-z47	2	3	3	2	4	1
6-z47i	2	3	3	2	4	1
6-z48	2	3	2	3	4	1
6-z49	2	2	4	3	2	2
6-z50	2	2	4	2	3	2

7-1	6	5	4	3	2	1
7-2	5	5	4	3	3	1
7-2i	5	5	4	3	3	1
7-3	5	4	4	4	3	1
7-3i	5	4	4	4	3	1
7-4	5	4	4	3	3	2
7-4i	5	4	4	3	3	2
7-5	5	4	3	3	4	2
7-5i	5	4	3	3	4	2
7-6	5	3	3	4	4	2
7-6i	5	3	3	4	4	2
7-7	5	3	2	3	5	3
7-7i	5	3	2	3	5	3
7-8	4	5	4	4	2	2
7-9	4	5	3	4	3	2
7-9i	4	5	3	4	3	2
7-10	4	4	5	3	3	2
7-10i	4	4	5	3	3	2
7-11	4	4	4	4	4	1
7-11i	4	4	4	4	4	1
7-z12	4	4	4	3	4	2
7-13	4	4	3	5	3	2

7-13i	4	4	3	5	3	2
7-14	4	4	3	3	5	2
7-14i	4	4	3	3	5	2
7-15	4	4	2	4	4	3
7-16	4	3	5	4	3	2
7-16i	4	3	5	4	3	2
7-z17	4	3	4	5	4	1
7-z18	4	3	4	4	4	2
7-z18i	4	3	4	4	4	2
7-19	4	3	4	3	4	3
7-19i	4	3	4	3	4	3
7-20	4	3	3	4	5	2
7-20i	4	3	3	4	5	2
7-21	4	2	4	6	4	1
7-21i	4	2	4	6	4	1
7-22	4	2	4	5	4	2
7-23	3	5	4	3	5	1
7-23i	3	5	4	3	5	1
7-24	3	5	3	4	4	2
7-24i	3	5	3	4	4	2
7-25	3	4	5	3	4	2
7-25i	3	4	5	3	4	2
7-26	3	4	4	5	3	2
7-26i	3	4	4	5	3	2
7-27	3	4	4	4	5	1
7-27i	3	4	4	4	5	1
7-28	3	4	4	4	3	3
7-28i	3	4	4	4	3	3
7-29	3	4	4	3	5	2
7-29i	3	4	4	3	5	2
7-30	3	4	3	5	4	2
7-30i	3	4	3	5	4	2
7-31	3	3	6	3	3	3
7-31i	3	3	6	3	3	3
7-32	3	3	5	4	4	2
7-32i	3	3	5	4	4	2
7-33	2	6	2	6	2	3
7-34	2	5	4	4	4	2
7-35	2	5	4	3	6	1
7-z36	4	4	4	3	4	2
7-z36i	4	4	4	3	4	2
7-z37	4	3	4	5	4	1
7-z38	4	3	4	4	4	2
7-z38i	4	3	4	4	4	2

8-1	7	6	5	4	4	2
8-2	6	6	5	5	4	2
8-2i	6	6	5	5	4	2
8-3	6	5	6	5	4	2
8-4	6	5	5	5	5	2
8-4i	6	5	5	5	5	2
8-5	6	5	4	5	5	3
8-5i	6	5	4	5	5	3
8-6	6	5	4	4	6	3
8-7	6	4	5	6	5	2
8-8	6	4	4	5	6	3
8-9	6	4	4	4	6	4
8-10	5	6	6	4	5	2
8-11	5	6	5	5	5	2
8-11i	5	6	5	5	5	2
8-12	5	5	6	5	4	3
8-12i	5	5	6	5	4	3
8-13	5	5	6	4	5	3
8-13i	5	5	6	4	5	3

8-14	5	5	5	5	6	2
8-14i	5	5	5	5	6	2
8-z15	5	5	5	5	5	3
8-z15i	5	5	5	5	5	3
8-16	5	5	4	5	6	3
8-16i	5	5	4	5	6	3
8-17	5	4	6	6	5	2
8-18	5	4	6	5	5	3
8-18i	5	4	6	5	5	3
8-19	5	4	5	7	5	2
8-19i	5	4	5	7	5	2
8-20	5	4	5	6	6	2
8-21	4	7	4	6	4	3
8-22	4	6	5	5	6	2
8-22i	4	6	5	5	6	2
8-23	4	6	5	4	7	2
8-24	4	6	4	7	4	3
8-25	4	6	4	6	4	4
8-26	4	5	6	5	6	2
8-27	4	5	6	5	5	3
8-27i	4	5	6	5	5	3
8-28	4	4	8	4	4	4
8-z29	5	5	5	5	5	3
8-z29i	5	5	5	5	5	3

9-1	8	7	6	6	6	3
9-2	7	7	7	6	6	3
9-2i	7	7	7	6	6	3
9-3	7	6	7	7	6	3
9-3i	7	6	7	7	6	3
9-4	7	6	6	7	7	3
9-4i	7	6	6	7	7	3
9-5	7	6	6	6	7	4
9-5i	7	6	6	6	7	4
9-6	6	8	6	7	6	3
9-7	6	7	7	6	7	3
9-7i	6	7	7	6	7	3
9-8	6	7	6	7	6	4
9-8i	6	7	6	7	6	4
9-9	6	7	6	6	8	3
9-10	6	6	8	6	6	4
9-11	6	6	7	7	7	3
9-11i	6	6	7	7	7	3
9-12	6	6	6	9	6	3

10-1	9	8	8	8	8	4
10-2	8	9	8	8	8	4
10-3	8	8	9	8	8	4
10-4	8	8	8	9	8	4
10-5	8	8	8	8	9	4
10-6	8	8	8	8	8	5
10-1	9	8	8	8	8	4
10-2	8	9	8	8	8	4
10-3	8	8	9	8	8	4
10-4	8	8	8	9	8	4
10-5	8	8	8	8	9	4
10-6	8	8	8	8	8	5

11-1	10	10	10	10	10	5

12-1	11	11	11	11	11	6

Appendix 3
Proof of CIG-extension rules

Preliminary remarks:

- CIG-3 with notes a, b and c (in that order) is written as CIG-3 (a-b-c).[588]

- The interval class between two notes a and b is written as $ic_{(a-b)}$.

- ICV is the "interval class vector rule": "The number of ic 1's (the first number) in the interval class vectors of any CIG-n is n-1 for instances of set classes with forte number [n-1], and n-2 for all other CIG-n's. The ic-vectors of all other set classes have a number of ic 1's lower than n-2".

- Numbers between brackets at the right end of each line in the proof refer to the lines from which the line in question is deduced.

Extension rule for CIG-3's:

If CIG-3 (a-b-c) is extended by adding a note p that is the last note of a unique CIG-3 (b-c-p), the resulting 4-note group (a-b-c-p) is always a CIG-4 or a 4-note CIG-3, except in some cases where $ic_{(b-c)}$ = 1 and (a-b-c) is no instance of set class [3-1].

Proof:

1.0 If p = a ⇒ no new pitch class is added and (a-b-c-p) is a 4-note CIG-3.
QED

2.0 If p ≠ a ⇒ (a-b-c-p) is a permutation of an instance of a set class of cardinality 4. Two possible cases are considered:

Case 1: $ic_{(b-c)} \neq 1$

1.1	$ic_{(b-c)} \neq 1$	
1.2	(b-c-p) is a CIG-3	
1.3	⇒The ic 1-value in the ic-vector of (b-c-p) ≠ 0	(1.2 & ICV)
1.4	⇒ either $ic_{(b-p)} = 1$ or $ic_{(c-p)} = 1$ (or both)	(1.1 & 1.3)
1.5	(a-b-c) is a CIG-3	
1.6	⇒The ic 1-value in the ic-vector of (a-b-c) ≠ 0	(1.5 & ICV)
1.7	⇒ either $ic_{(a-b)} = 1$ or $ic_{(a-c)} = 1$ (or both)	(1.1 & 1.6)
1.8	⇒ the ic 1-value in the ic-vector of (a-b-c-p) is at least 2	(1.4 & 1.7)
1.9	⇒ (a-b-c-p) is a CIG-4	(1.8 & ICV)

QED

[588] a, b, and c are not pitch names (which are always written in capitals), but letter names indicating the position of the pitch classes within the note group. They may represent any pitch classes.

Case 2: $ic_{(b-c)} = 1$

Case 2.1: (a-b-c) is no instance of [3-1]

2.1.1	$ic_{(b-c)} = 1$	
2.1.2	(a-b-c) is a CIG, but no instance of [3-1]	
2.1.3	⇒ the ic 1-value in the ic-vector of (a-b-c) = 1	(2.1.2 & ICV)
2.1.4	⇒ $ic_{(a-b)} \neq 1$ and $ic_{(a-c)} \neq 1$	(2.1.1 & 2.1.3)
2.1.5	(b-c-p) is a CIG-3	
2.1.6	$ic_{(c-p)}$ can be any number between 1 and 6	(2.1.1 & 2.1.5)
2.1.7	(a-b-c-p) is an instance of a set class of cardinality 4	
2.1.8	(a-b-c-p) is a CIG-4 if the ic 1-value in its ic-vector is at least 2	(ICV)
2.1.9	⇒ (a-b-c-p) is only a CIG-4 if $ic_{(a-p)} = 1$, or $ic_{(b-p)} = 1$, or $ic_{(c-p)} = 1$	(2.1.1, 2.1.4, 2.1.7 & 2.1.8)
	QED	

Case 2.2: (a-b-c) is an instance of [3-1]

2.2.1	$ic_{(b-c)} = 1$	
2.2.2	(a-b-c) is an instance of [3-1]	
2.2.3	⇒ the ic 1-value in the ic-vector of (a-b-c) = 2	(2.2.2 & ICV)
2.2.4	⇒ $ic_{(a-b)} = 1$ or $ic_{(a-c)} = 1$ (but not both)	(2.2.1 & 2.2.2)
2.2.5	(a-b-c-p) is a superset of (a-b-c)	
2.2.6	⇒ the ic 1-value in the ic-vector of (a-b-c-p) is at least 2	(2.2.3, & 2.2.5)
2.2.7	(a-b-c-p) is a CIG-4	(2.2.6, & ICV)
	QED	

Extension rule for CIG-4's:

If CIG-4 (a-b-c-d) consisting of two unique CIG-3's (a-b-c) and (b-c-d) is extended by adding a note p that is the last note of unique CIG-3 (c-d-p) and of CIG-4 (b-c-d-p), the resulting 5-note group (a-b-c-d-p) is always a CIG-5 or a 5-note CIG-4, except in some cases where (b-c-d) is an instance of 3-1 with $ic_{(b-c)} = 1$ and $ic_{(c-d)} = 1$ (and both +1 or both -1)[589].

Proof:

1.0 If p = a ⇒ no new pitch class is added and (a-b-c-d-p) is a 5-note CIG-4.
QED

2.0 If p ≠ a ⇒ (a-b-c-d-p) is a permutation of an instance of a set class of cardinality 5. Two possible cases are considered:

Case 1: $ic_{(c-d)} \neq 1$

1.1	$ic_{(c-d)} \neq 1$	
1.2	(c-d-p) is a CIG-3	
1.3	⇒The ic 1-value in the ic-vector of (c-d-p) ≠ 0	(1.2 & ICV)
1.4	⇒ either $ic_{(c-p)} = 1$ or $ic_{(d-p)} = 1$ (or both)	(1.1 & 1.3)
1.5	(a-b-c-d) is a CIG-4	
1.6	⇒ the ic 1-value in the ic-vector of (a-b-c-d) = 2 or 3	(1.5 & ICV)

589 Otherwise note b = note d and (b-c-d) is no CIG-3.

1.7	(a-b-c-d-p) is a superset of (a-b-c-d)	
1.8	⇒ the ic 1-value in the ic-vector of (a-b-c-d-p) = 3 or 4	(1.4, 1.6 & 1.7)
1.9	⇒ (a-b-c-d-p) is a CIG-5	(1.8 & ICV)
	QED	

Case 2: $ic_{(c\text{-}d)} = 1$

Case 2.1: (b-c-d) is no instance of [3-1]

2.1.1	$ic_{(c\text{-}d)} = 1$	
2.1.2	CIG (b-c-d) is no instance of [3-1]	
2.1.3	⇒ the ic 1-value in the ic-vector of (b-c-d) = 1	(2.1.2 & ICV)
2.1.4	⇒ $ic_{(b\text{-}c)} \neq 1$ and $ic_{(b\text{-}d)} \neq 1$	(2.1.1 & 2.1.3)
2.1.5	(b-c-d-p) is a CIG-4	
2.1.6	⇒ the ic 1-value in the ic-vector of (b-c-d-p) = 2 or 3	(2.1.5 & ICV)
2.1.7	⇒ $ic_{(b\text{-}p)} = 1$, or $ic_{(c\text{-}p)} = 1$, or $ic_{(d\text{-}p)} = 1$	(2.1.1, 2.1.4 & 2.1.6)
2.1.8	(a-b-c) is a CIG-3	
2.1.9	⇒the ic 1-value in the ic-vector of (a-b-c) ≠ 0	(2.1.8 & ICV)
2.1.10	⇒ either $ic_{(a\text{-}b)} = 1$ or $ic_{(a\text{-}c)} = 1$	(2.1.4 & 2.1.9)
2.1.11	⇒ the ic 1-value in the ic-vector of (a-b-c-d-p) = 3 or 4	(2.1.6 & 2.1.10)
2.1.12	⇒ (a-b-c-d-p) is a CIG-5	(2.1.11 & ICV)
	QED	

Case 2.2: (b-c-d) is an instance of [3-1]

Case 2.2.1: $ic_{(b\text{-}d)} = 1$

2.2.1.1	$ic_{(b\text{-}d)} = 1$ and $ic_{(c\text{-}d)} = 1$	
2.2.1.2	(b-c-d) is an instance of [3-1]	
2.2.1.3	⇒ the ic 1-value in the ic-vector of (b-c-d) = 2	(2.2.1.2 & ICV)
2.2.1.4	⇒ $ic_{(b\text{-}c)} \neq 1$	(2.2.1.1 & 2.2.1.3)
2.2.1.5	(a-b-c) is a CIG-3	
2.2.1.6	⇒the ic 1-value in the ic-vector of (a-b-c) ≠ 0	(2.2.1.5 & ICV)
2.2.1.7	⇒ either $ic_{(a\text{-}b)} = 1$ or $ic_{(a\text{-}c)} = 1$	(2.2.1.4 & 2.2.1.6)
2.2.1.8	⇒ the ic 1-value in the ic-vector of (a-b-c-d) = 3	(2.2.1.1 & 2.2.1.7)
2.2.1.9	(a-b-c-d-p) is a superset of (a-b-c-d)	
2.2.1.10	⇒ the ic 1-value in the ic-vector of (a-b-c-d-p) ≥ 3	(2.2.1.8 & 2.2.1.9)
2.2.1.11	⇒ (a-b-c-d-p) in a CIG-5	(2.2.1.10 & ICV)
	QED	

Case 2.2.2: $ic_{(b\text{-}c)} = 1$

2.2.2.1	$ic_{(b\text{-}c)} = 1$ and $ic_{(c\text{-}d)} = 1$	
2.2.2.2	(b-c-d) is an instance of [3-1]	
2.2.2.3	⇒ the ic 1-value in the ic-vector of (b-c-d) = 2	(2.2.2.2 & ICV)
2.2.2.4	⇒ $ic_{(b\text{-}d)} \neq 1$	(2.2.2.1)
2.2.2.5	(a-b-c) is a CIG	
2.2.2.6	⇒ note a ≠ note c	(2.2.2.5)
2.2.2.7	⇒ note a ≠ note b	(2.2.2.5)
2.2.2.8	(a-b-c) & (b-c-d) are unique CIG's	
2.2.2.9	⇒ $ic_{(a\text{-}b)} \neq 1$	(2.2.2.1, 2.2.2.6 & 2.2.2.8)
2.2.2.10	(a-b-c-d) is a CIG-4	

2.2.2.11 ⇒ note a ≠ note d (2.2.2.11)
2.2.2.12 ⇒ $ic_{(a-c)} \neq 1$ (2.2.2.1, 2.2.2.7, 2.2.2.9 & 2.2.2.11)

Case 2.2.2.1: (a-b-c-d) is an instance of [4-1]

2.2.2.1.1 (a-b-c-d) is an instance of [4-1]
2.2.2.1.2 ⇒ the ic 1-value in the ic-vector of (a-b-c-d) = 3 (2.2.2.1.1 & ICV)
2.2.2.1.3 (a-b-c-d-p) is a superset of (a-b-c-d)
2.2.2.1.4 ⇒ the ic 1-value in the ic-vector of (a-b-c-d-p) ≥ 3 (2.2.2.1.2 & 2.2.2.1.3)
2.2.2.1.5 ⇒ (a-b-c-d-p) in a CIG-5 (2.2.2.1.4 & ICV)
QED

Case 2.2.2.2: (a-b-c-d) is no instance of [4-1]

2.2.2.2.1 CIG (a-b-c-d) is no instance of [4-1]
2.2.2.2.2 ⇒ the ic 1-value in the ic-vector of (a-b-c-d) = 2 (2.2.2.1.1 & ICV)
2.2.2.2.3 (a-b-c-d-p) is a CIG-5 if ic 1-value in its ic-vector ≥3 (ICV)
2.2.2.2.4 ⇒ (a-b-c-p) is only a CIG-5 if $ic_{(a-p)} = 1$, or $ic_{(b-p)} = 1$ (2.2.2.1.2 & 2.2.2.1.3)
QED

Extension rule for CIG-n's, for 5 ≤ n:

If CIG-n (for 5 ≤ n) (z(n)-z(n-1)…z(2)-z(1)), consisting exclusively of unique CIG's of all cardinalities between 3 and n, is extended by adding a note p that is the last note of unique CIG-3 (z(2)-z(1)-p), of CIG-4 (z(3)-z(2)-z(1)-p), and of CIG-5 (z(4)-z(3)-z(2)-z(1)-p), the resulting (n+1)-note group (z(n)-z(n-1)…z(2)-z(1)-p) is always a CIG-(n+1) or a (n+1)-note CIG-n.

Proof:

1.0 (z(4)-z(3)-z(2)-z(1)-p) is a CIG-5 ⇒ for n ≤ 4, p ≠ z(n)

2.0 if p = z(n) for some n > 4 ⇒ no new pitch class is added, and (z(n)-z(n-1)…z(2)-z(1)-p) is an (n+1)-note CIG-n
QED

3.0 if p ≠ z(n) for all n > 4 ⇒ (z(n)-z(n-1)…z(2)-z(1)-p) is an (n+1)-note group. A distinction is made between $ic_{(z(2)-z(1))} \neq 1$ (Case 1) and $ic_{(z(2)-z(1))} = 1$ (Case 2):

Case 1: $ic_{(z(2)-z(1))} \neq 1$

1.1 $ic_{(z(2)-z(1))} \neq 1$
1.2 (z(2)-z(1)-p) is a CIG-3
1.3 ⇒The ic 1-value in the ic-vector of (z(2)-z(1)-p) ≠ 0 (1.2 & ICV)
1.4 ⇒ $ic_{(z(2)-p)} = 1$ or $ic_{(z(1)-p)} = 1$ (or both) (1.1 & 1.3)
1.5 (z(n)-z(n-1)…z(2)-z(1)) is a CIG-n
1.6 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 or n-2 (1.5 & ICV)
1.7 (z(n)-z(n-1)…z(2)-z(1)-p) is a superset of (z(n)-z(n-1)…z(2)-z(1))
1.8 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n or n-1 (1.4, 1.6 & 1.7)

1.9 ⇒ (z(n)-z(n-1)…z(2)-z(1)) is a CIG-(n+1) (1.8 & ICV)
QED

Case 2: $ic_{(z(2)-z(1))} = 1$

Case 2.1: (z(3)-z(2)-z(1)) is no instance of set class [3-1]

2.1.1 $ic_{(z(2)-z(1))} = 1$
2.1.2 CIG-3 (z(3)-z(2)-z(1)) is no instance of set class [3-1]
2.1.3 ⇒ the ic 1-value in the ic-vector of (z(3)-z(2)-z(1)) = 1 (2.1.2 & ICV)
2.1.4 ⇒ $ic_{(z(3)-z(2))} \neq 1$ and $ic_{(z(3)-z(1))} \neq 1$ (2.1.1 & 2.1.3)
2.1.5 (z(3)-z(2)-z(1)-p) is a CIG-4
2.1.6 ⇒ the ic 1-value in the ic-vector of (z(3)-z(2)-z(1)-p) = 2 or 3 (2.1.5 & ICV)
2.1.7 ⇒ $ic_{(z(3)-p)} = 1$, or $ic_{(z(2)-p)} = 1$, or $ic_{(z(1)-p)} = 1$ (2.1.1, 2.1.4 & 2.1.6)
2.1.8 (z(n)-z(n-1)…z(2)-z(1)) is a CIG-n
2.1.9 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 or n-2 (2.1.8 & ICV)
2.1.10 (z(n)-z(n-1)…z(2)-z(1)-p) is a superset of (z(n)-z(n-1)-…-z(3)-z(2)-z(1))
2.1.11 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)-p) = n or n-1 (2.1.7, 2.1.9 & 2.1.10)
2.1.12 ⇒ (z(n)-z(n-1)…z(2)-z(1)-p) is a CIG-(n+1) (2.1.11 & ICV)
QED

Case 2.2: (z(3)-z(2)-z(1)) is an instance of set class [3-1]

Case 2.2.1: $ic_{(z(3)-z(1))} \neq 1$

2.2.1.1 $ic_{(z(3)-z(1))} \neq 1$ and $ic_{(z(2)-z(1))} = 1$
2.2.1.2 GIG-3 (z(3)-z(2)-z(1)) is an instance of [3-1]
2.2.1.3 ⇒ the ic 1-value in the ic-vector of (z(3)-z(2)-z(1)) = 2 (2.2.1.2 & ICV)
2.2.1.4 ⇒ $ic_{(z(3)-z(2))} = 1$ (2.2.1.1 & 2.2.1.3)

Case 2.2.1.1: (z(4)-z(3)-z(2)-z(1)) is no instance of [4-1]

2.2.1.1.1 CIG-4 (z(4)-z(3)-z(2)-z(1)) is no instance of [4-1]
2.2.1.1.2 ⇒ the ic 1-value in the ic-vector of (z(4)-z(3)-z(2)-z(1)) = 2 (2.2.1.1.1 & ICV)
2.2.1.1.3 $ic_{(z(3)-z(2))} = 1$, $ic_{(z(2)-z(1))} = 1$, and $ic_{(z(3)-z(1))} \neq 1$
2.2.1.1.4 ⇒ $ic_{(z(4)-z(3))} \neq 1$, $ic_{(z(4)-z(2))} \neq 1$ and $ic_{(z(4)-z(1))} \neq 1$ (2.2.1.1.2, & 2.2.1.1.3)
2.2.1.1.5 (z(4)-z(3)-z(2)-z(1)-p) is a CIG-5
2.2.1.1.6 ⇒ the ic 1-value in the ic-vector of (z(4)-z(3)-z(2)-z(1)-p) = 3 or 4 (2.2.1.1.5 & ICV)
2.2.1.1.7 if $ic_{(z(1)-p)} = 1$ ⇒ note p = note z(2) and then (z(2)-z(1)-p) is no CIG-3, or (z(3)-z(2)-z(1)) = (z(2)-z(1)-p) (2.2.1.2 & 2.2.1.1.3)
2.2.1.1.8 ⇒ $ic_{(z(1)-p)} \neq 1$ (2.2.1.1.7)
2.2.1.1.9 ⇒ $ic_{(z(4)-p)} = 1$, or $ic_{(z(3)-p)} = 1$, or $ic_{(z(2)-p)} = 1$ (2.2.1.1.3, 2.2.1.1.4, 2.2.1.1.6 & 2.2.1.1.8)
2.2.2.1.10 (z(n)-z(n-1)…z(2)-z(1)) is a CIG-n
2.2.2.1.11 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 or n-2 (2.2.1.1.10 & ICV)
2.2.2.1.12 (z(n)-z(n-1)…z(2)-z(1)-p) is a superset of (z(n)-z(n-1)…z(2)-z(1))
2.2.2.1.13 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)-p) = n or n-1 (2.2.1.1.9, 2.2.1.1.11 & 2.2.1.1.12)

2.2.2.1.14 ⇒ (z(n)-z(n-1)…z(2)-z(1)-p) is a CIG-(n+1) (2.2.1.1.13 & ICV)
QED

<u>Case 2.2.1.2</u>: (z(4)-z(3)-z(2)-z(1)) is an instance of [4-1]

2.2.1.2.1 (z(4)-z(3)-z(2)-z(1)) is an instance of [4-1]
2.2.1.2.2 ⇒ the ic 1-value in the ic-vector of (z(4)-z(3)-z(2)-z(1)) = 3 (2.2.2.1.1 & ICV)
2.2.1.2.3 $ic_{(z(3)-z(2))} = 1$ and $ic_{(z(2)-z(1))} = 1$
2.2.1.2.4 if $ic_{(z(4)-z(3))} = 1$ ⇒ note z(4) = note z(2) and (z(4)-z(3)-z(2)) is no CIG-3, or (z(4)-z(3)-z(2)) = (z(3)-z(2)-z(1)) (2.2.1.2.1 & 2.2.1.2.3)
2.2.1.2.5 ⇒ $ic_{(z(4)-z(3))} \neq 1$ (2.2.1.2.4)
2.2.1.2.6 if $ic_{(z(4)-z(2))} = 1$ ⇒ note z(4) = note z(3) and (b-c-d) is no CIG-3, or note z(4) = note z(1) and (z(4)-z(3)-z(2)-z(1)) is no CIG-4. (2.2.1.2.1 & 2.2.1.2.3)
2.2.1.2.7 ⇒ $ic_{(z(4)-z(2))} \neq 1$ (2.2.1.2.6)
2.2.2.2.8 $ic_{(z(3)-z(1))} \neq 1$
2.2.1.2.9 ⇒ $ic_{(z(4)-z(2))} = 1$ (2.2.1.2.2, 2.2.1.2.3, 2.2.1.2.5, 2.2.1.2.7 & 2.2.1.2.8)
2.2.1.2.10 (z(5)-z(4)-z(3)) is a CIG-3
2.2.1.2.11 ⇒ the ic 1-value in the ic-vector of (z(5)-z(4)-z(3)) ≠ 0 (2.2.1.2.10 & ICV)
2.2.1.2.12 ⇒ $ic_{(z(5)-z(4))} = 1$ or $ic_{(z(5)-z(3))} = 1$ (2.2.2.1.5 & 2.2.1.2.11)
2.2.1.2.13 (z(5)-z(4)-z(3)-z(2)-z(1)) is a superset of (z(4)-z(3)-z(2)-z(1))
2.2.1.2.14 ⇒ the ic 1-value in the ic-vector of (z(5)-z(4)-z(3)-z(2)-z(1)) = 4 (2.2.1.2.2, 2.2.1.2.12 & 2.2.1.2.13)
2.2.1.2.15 ⇒ (z(5)-z(4)-z(3)-z(2)-z(1)) in an instance of [5-1] (2.2.1.2.14 & ICV)
go to 4.0

<u>Case 2.2.2</u>: $ic_{(z(3)-z(1))} = 1$

2.2.2.1 $ic_{(z(3)-z(1))} = 1$
2.2.2.2 $ic_{(z(2)-z(1))} = 1$
2.2.2.3 (z(3)-z(2)-z(1)) is an instance of set class [3-1]
2.2.2.4 ⇒ the ic 1-value in the ic-vector of (z(3)-z(2)-z(1)) = 2 (2.2.2.13 & ICV)
2.2.2.5 ⇒ $ic_{(z(3)-z(2))} \neq 1$ (2.2.2.1, 2.2.2.2 & 2.2.2.4)
2.2.2.6 (z(4)-z(3)-z(2)) is a CIG-3
2.2.2.7 ⇒The ic 1-value in the ic-vector of (z(4)-z(3)-z(2)) ≠ 0 (2.2.2.6 & ICV)
2.2.2.8 ⇒ $ic_{(z(4)-z(3))} = 1$ or $ic_{(z(4)-z(2))} = 1$ (2.2.2.5 & 2.2.2.7)
2.2.2.9 ⇒ the ic 1-value in the ic-vector of (z(4)-z(3)-z(2)-z(1)) = 3 (2.2.2.1, 2.2.2.2 & 2.2.2.8)
2.2.2.10 (z(4)-z(3)-z(2)-z(1)) is an instance of [4-1]
⇒ $ic_{(z(4)-z(1))} \neq 1$ (2.2.2.1, 2.2.2.2, 2.2.2.8 & 2.2.2.9)

<u>Case 2.2.2.1</u>: $ic_{(z(4)-z(3))} = 1$

2.2.2.1.1 $ic_{(z(4)-z(3))} = 1$
2.2.2.1.2 ⇒ $ic_{(z(4)-z(2))} \neq 1$ (2.2.2.1, 2.2.2.2, 2.2.2.9 & 2.2.2.1.1)
2.2.2.1.3 if $ic_{(z(5)-z(3))} = 1$ ⇒ note z(5) = note z(4), or note z(5) = note z(1)
⇒ (z(5)-z(4)-z(3)-z(2)-z(1)) is no CIG-5. (2.2.2.1, 2.2.2.2 & 2.2.2.1.1)
2.2.2.1.4 ⇒ $ic_{(z(5)-z(3))} \neq 1$ (2.2.2.1.3)
2.2.2.1.5 (z(5)-z(4)-z(3)-z(2)) is CIG-4
2.2.2.1.6 ⇒ the ic 1-value in the ic-vector of (z(5)-z(4)-z(3)-z(2)) = 2 or 3 (2.2.2.1.5 & ICV)
2.2.2.1.7 ⇒ $ic_{(z(5)-z(2))} = 1$ or $ic_{(z(5)-z(4))} = 1$ (2.2.2.5, 2.2.2.1.1, 2.2.2.1.2, 2.2.2.1.4 & 2.2.2.1.6)
2.2.2.1.8 (z(5)-z(4)-z(3)-z(2)-z(1)) is a superset of (z(4)-z(3)-z(2)-z(1))

2.2.2.1.9 ⇒ the ic 1-value in the ic-vector of (z(5)-z(4)-z(3)-z(2)-z(1)) = 4 (2.2.2.9, 2.2.2.1.7 & 2.2.2.1.8)

2.2.2.1.10 ⇒ (z(5)-z(4)-z(3)-z(2)-z(1)) in an instance of [5-1] (2.2.2.1.9 & ICV)
go to 4.0

Case 2.2.2.2: $ic_{(z(4)-z(3))} \neq 1$

2.2.2.2.1 $ic_{(z(4)-z(3))} \neq 1$
2.2.2.2.2 ⇒ $ic_{(z(4)-z(2))} = 1$ (2.2.2.1, 2.2.2.2, 2.2.2.9 & 2.2.2.2.1)
2.2.2.2.3 (z(5)-z(4)-z(3)) is a CIG-3
2.2.2.2.4 ⇒The ic 1-value in the ic-vector of (z(5)-z(4)-z(3)) $\neq 0$ (2.2.2.2.4 & ICV)
2.2.2.2.5 $ic_{(z(5)-z(4))} = 1$ or $ic_{(z(5)-z(3))} = 1$ (2.2.2.2.1 & 2.2.2.2.4)
2.2.2.2.6 (z(5)-z(4)-z(3)-z(2)-z(1)) is a superset of (z(4)-z(3)-z(2)-z(1))
2.2.2.2.7 ⇒ the ic 1-value in the ic-vector of (z(5)-z(4)-z(3)-z(2)) = 4 (2.2.2.9, 2.2.2.2.5 & 2.2.2.2.6)
2.2.2.2 ⇒ (z(5)-z(4)-z(3)-z(2)-z(1)) in an instance of [5-1] (2.2.2.2.7 & ICV)
go to 4.0

4.0 for n = 4: if (z(4)-z(3)-z(2)-z(1)) is an instance of [4-1], if (z(3)-z(2)-z(1)) is an instance of [3-1], and if $ic_{(z(2)-z(1))} = 1$⇒ (z(5)-z(4)-z(3)-z(2)-z(1)) in an instance of 5-1 (Case 2.2.1.2 and Case 2.2.2)

4.1 assume: (z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [n-1] for $4 \leq n$,
and (z(n-1)…z(2)-z(1)) is an instance of set class [(n-1)-1] for $3 \leq n$
and (z(n-2)…-z(1)) is an instance of set class [(n-2)-1] for $2 \leq n$

4.2 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 (4.1 & ICV)

4.3 (z(n+1)-z(n)-z(n-1)) is a CIG-3

4.4 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)) $\neq 0$ (4.3 & ICV)

Case 4.1: $ic_{(z(n+1)-z(n))} = 1$

4.1.1 $ic_{(z(n+1)-z(n))} = 1$
4.1.2 (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is a superset of (z(n)-z(n-1)…z(2)-z(1))
4.1.3 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) = n (4.2 & 4.1.1)
4.1.4 ⇒ (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [(n+1)-1] (4.1.3 & ICV)
QED

Case 4.2: $ic_{(z(n+1)-z(n))} \neq 1$

4.2.1 $ic_{(z(n+1)-z(n))} \neq 1$
4.2.2 ⇒ $ic_{(z(n+1)-z(n-1))} = 1$ or $ic_{(z(n)-z(n-1))} = 1$ (4.4 & 4.2.1)

Case 4.2.1: $ic_{(z(n+1)-z(n-1))} = 1$

4.2.1.1 $ic_{(z(n+1)-z(n-1))} = 1$
4.2.1.2 (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is a superset of (z(n)-z(n-1)…-z(2)-z(1))

4.2.1.3 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) = n (4.2 & 4.2.1.1)

4.2.1.4 ⇒ (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [(n+1)-1] (4.2.1.3 & ICV)

QED

<u>Case 4.2.2</u>: $ic_{(z(n+1)-z(n-1))} \neq 1$

4.2.2.1 $ic_{(z(n+1)-z(n-1))} \neq 1$

4.2.2.2 ⇒ $ic_{(z(n)-z(n-1))} = 1$ (4.2.2 & 4.2.2.1)

4.2.2.3 (z(n+1)-z(n)-z(n-1)-z(n-2)) is a CIG-4

4.2.2.4 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)-z(n-2)) = 2 or 3 (4.2.2.3 & ICV)

4.2.2.5 if $ic_{(z(n)-z(n-2))} = 1$ ⇒ note z(n) = note z(n-1) and (z(n)-z(n-1)-z(n-2) is no CIG-3 (4.1 & 4.2.2.2)

4.2.2.6 ⇒ $ic_{(z(n)-z(n-2))} \neq 1$ (4.2.2.5)

4.2.2.7 ⇒ $ic_{(z(n+1)-z(n-2))} = 1$ or $ic_{(z(n-1)-z(n-2))} = 1$ (4.2.1, 4.2.2.1, 4.2.2.4 & 4.2.2.6)

<u>Case 4.2.2.1</u>: $ic_{(z(n+1)-z(n-2))} = 1$

4.2.2.1.1 $ic_{(z(n+1)-z(n-2))} = 1$

4.2.2.1.2 (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is a superset of (z(n)-z(n-1)…-z(2)-z(1))

4.2.2.1.3 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) = n (4.2 & 4.2.2.1.1)

4.2.2.1.4 ⇒ (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [(n+1)-1] (4.2.2.1.3 & ICV)

QED

<u>Case 4.2.2.2</u>: $ic_{(z(n+1)-z(n-2))} \neq 1$

4.2.2.2.1 $ic_{(z(n+1)-z(n-2))} \neq 1$

4.2.2.2.2 ⇒ $ic_{(z(n-1)-z(n-2))} = 1$ (4.2.2.7 & 4.2.2.2.1)

4.2.2.2.3 if $ic_{(z(n-2)-z(n-3))} = 1$ ⇒ (z(n)-z(n-1)-z(n-2)) = (z(n-1)-z(n-2)-z(n-3)) (4.2.2.2 & 4.2.2.2.2)

4.2.2.2.4 ⇒ $ic_{(z(n-2)-z(n-3))} \neq 1$ (4.2.2.2.3)

4.2.2.2.5 (z(n-2)…-z(1)) is an instance of set class [(n-2)-1]

4.2.2.2.6 ⇒ the ic 1-value in the ic-vector of (z(n-2)…-z(1)) = n-3 (4.2.2.2.5 & ICV)

4.2.2.2.7 (z(n-1)…-z(1)) is a superset of (z(n-2)…-z(1))

4.2.2.2.8 ⇒ $ic_{(z(n-1)-z(n-3))} \neq 1$ (4.2.2.2.2, 4.2.2.2.6 & 4.2.2.2.7)

4.2.2.2.8 (z(n-1)…z(2)-z(1)) is an instance of set class [(n-1)-1]

4.2.2.2.9 ⇒ the ic 1-value in the ic-vector of (z(n-1)…z(2)-z(1))= n-2 (4.2.2.2.8 & ICV)

4.2.2.2.10 ⇒ $ic_{(z(n)-z(n-3))} \neq 1$ (4.2.2.2, 4.2.2.2.6 & 4.2.2.2.9)

4.2.2.2.11 (z(n+1)-z(n)-z(n-1)-z(n-2)-z(n-3)) is a CIG-5

4.2.2.2.12 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)-z(n-2)-z(n-3)) = 3 or 4 (4.2.2.2.11 & ICV)

4.2.2.2.13 ⇒ $ic_{(z(n+1)-z(n-3))} = 1$
(4.2.2.1, 4.2.2.2, 4.2.2.6, 4.2.2.2.1, 4.2.2.2.2, 4.2.2.2.4, 4.2.2.2.8, 4.2.2.2.10 & 4.2.2.2.12)

4.2.2.2.14 (z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [n-1]

4.2.2.2.15 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 (4.2.2.2.14 & ICV)

4.2.2.2.16 (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is a superset of (z(n)-z(n-1)…-z(2)-z(1))

4.2.2.2.17 ⇒ the ic 1-value in the ic-vector of (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) = n

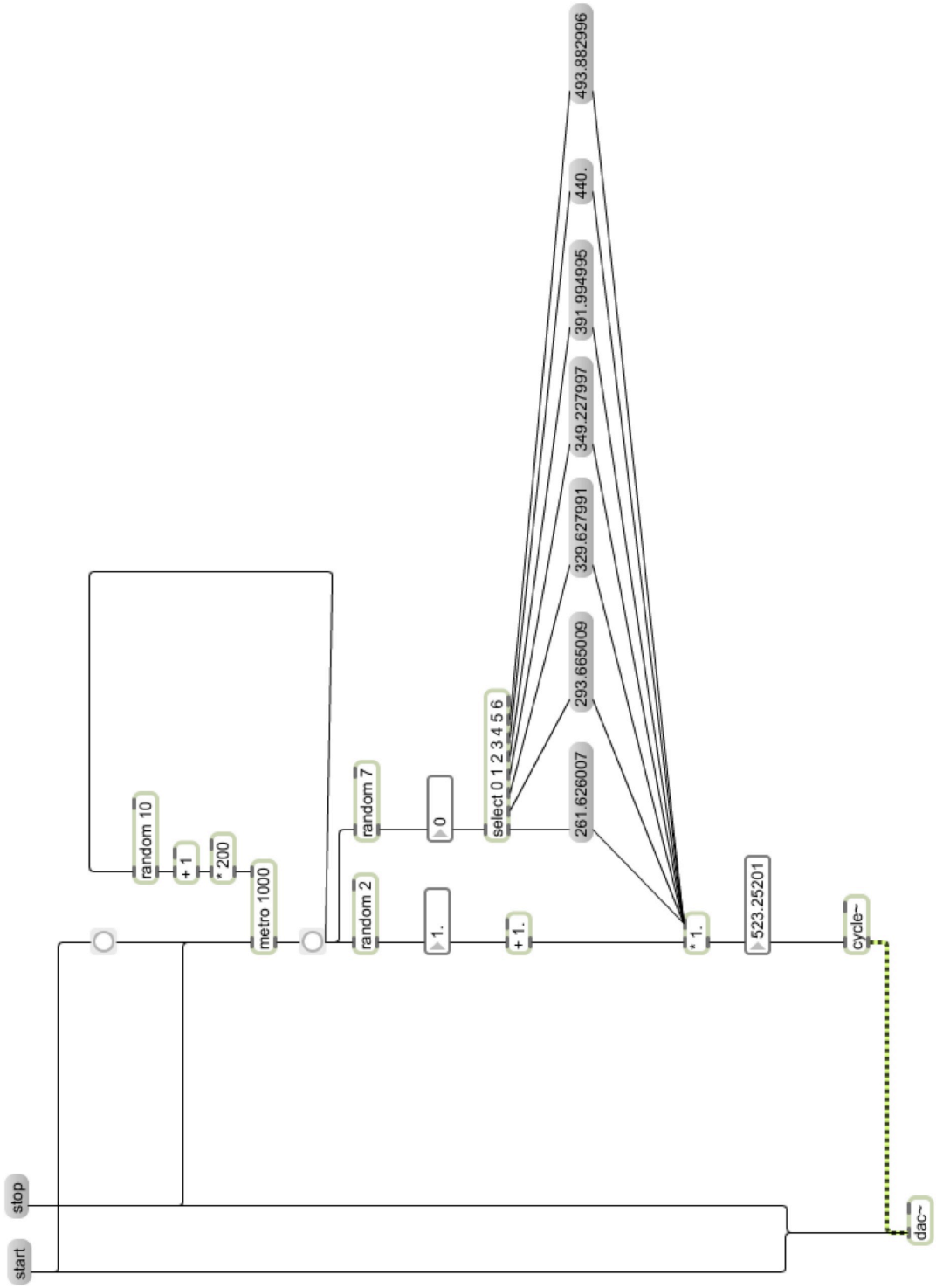
(4.2.2.2.13, 4.2.2.2.15 & 4.2.2.2.16)

4.2.2.2.18 ⇒ (z(n+1)-z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [(n+1)-1]
(4.2.2.2.13 & ICV)
QED

4.5 ⇒ (z(n)-z(n-1)…z(2)-z(1)) is an instance of set class [n-1] for $4 \leq n$
(4.0, 4.1, 4.1.4, 4.2.1.4, 4.2.2.1.4 & 4.2.2.2.18)
4.6 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)) = n-1 (4.5 & ICV)
4.7 (z(n)-z(n-1)…z(2)-z(1)-p) is a superset of (z(n)-z(n-1)…z(2)-z(1))
4.8 ⇒ the ic 1-value in the ic-vector of (z(n)-z(n-1)…z(2)-z(1)-p) ≥ n-1 (4.6 &4.7)
4.9 ⇒ (z(n)-z(n-1)…z(2)-z(1)-p) is a CIG-(n+1) (4.8 & ICV)
QED

Appendix 4
Max/MSP patch for Random Tonal Music Generator

(See Section 3.3.2)



Appendix 5
Polytonal variations

I composed three polytonal variations starting from the following four of Mark Delaere's six criteria for polytonal writing (see Section 3.6 and Mark Delaere, *'Autant de compositeurs, autant de polytonalités différentes': Polytonality in French Music Theory and Composition of the 1920s*), In Wörner, F., Scheideler, U., & Rupprecht, Ph. (eds.). *Tonality 1900-1950: Concept and Practice*, Franz Steiner Verlag, 2012, p. 163):

1. Use diatonic pitch material.
2. Combined keys should be related as remotely as possible
4. A polytonal composition preferably begins in one key, a second layer being added only later on.
6. Contrasting textures, rhythms, registers and instruments help to perceive tonal polarity.

The three variations consist of three parts performed by three instruments with contrasting tone colours: flute, gamba and piano (criterion 6). The three parts were composed independently, without any attention to the resulting harmonies. The scores are shown below.

In the first variation each instrument plays a part based on a single diatonic 7-set (7-35) without modulations (criterion 1). The flute uses the set of C major, the gamba the set of E major and the piano the set of A flat major. These sets are all a major third (ic 4) apart in order to obtain the least possible common notes between the sets (criterion 2) . Furthermore, in the first variation the three parts start at different moments in time (criterion 4) and they are written with contrasting rhythms in contrasting metres, textures and ranges (criterion 6): the flute plays a slurred melody in long notes in 4/4 time in a relatively high register; the gamba plays a legato ostinato rhythm in 5/8 metre consisting mainly of quavers with one group of two semi-quavers in the low register; the piano plays a rhythmically irregular staccato part with no perceivable metre, with very short note lengths, and abundant grace notes, and unpredictable accents in its middle register. The registers of the three parts are rigorously kept apart: the flute always playing the highest and the gamba always playing the lowest notes. As a result the three parts can be easily distinguished by the listeners, and each part is perceived as highly tonal.

In the second variation, the melodies and their distribution between the instruments is preserved, but the flute part is transposed an octave down and the piano part an octave up. This results in overlapping registers (all instruments play in the middle register of the piano), which weakens the effect of criterion 6. Indeed, the perception of three different tonalities in polytonal combination proofed to be harder than in Variation 1.

The third variation is composed with exactly the same material as Variation 2, but in this case the material is distributed freely among the three instruments. This variation is no longer perceived as polytonal since it is no longer possible to distinguish between the three simultaneous 'keys' . Variation 3 is therefore perceived as highly atonal. Whereas it is justified to consider the three parts separately in the T-analysis of Variation 1 (and to a lesser extent Variation 2), in a T-analysis of Variation 3 the three parts should be analysed as a whole.

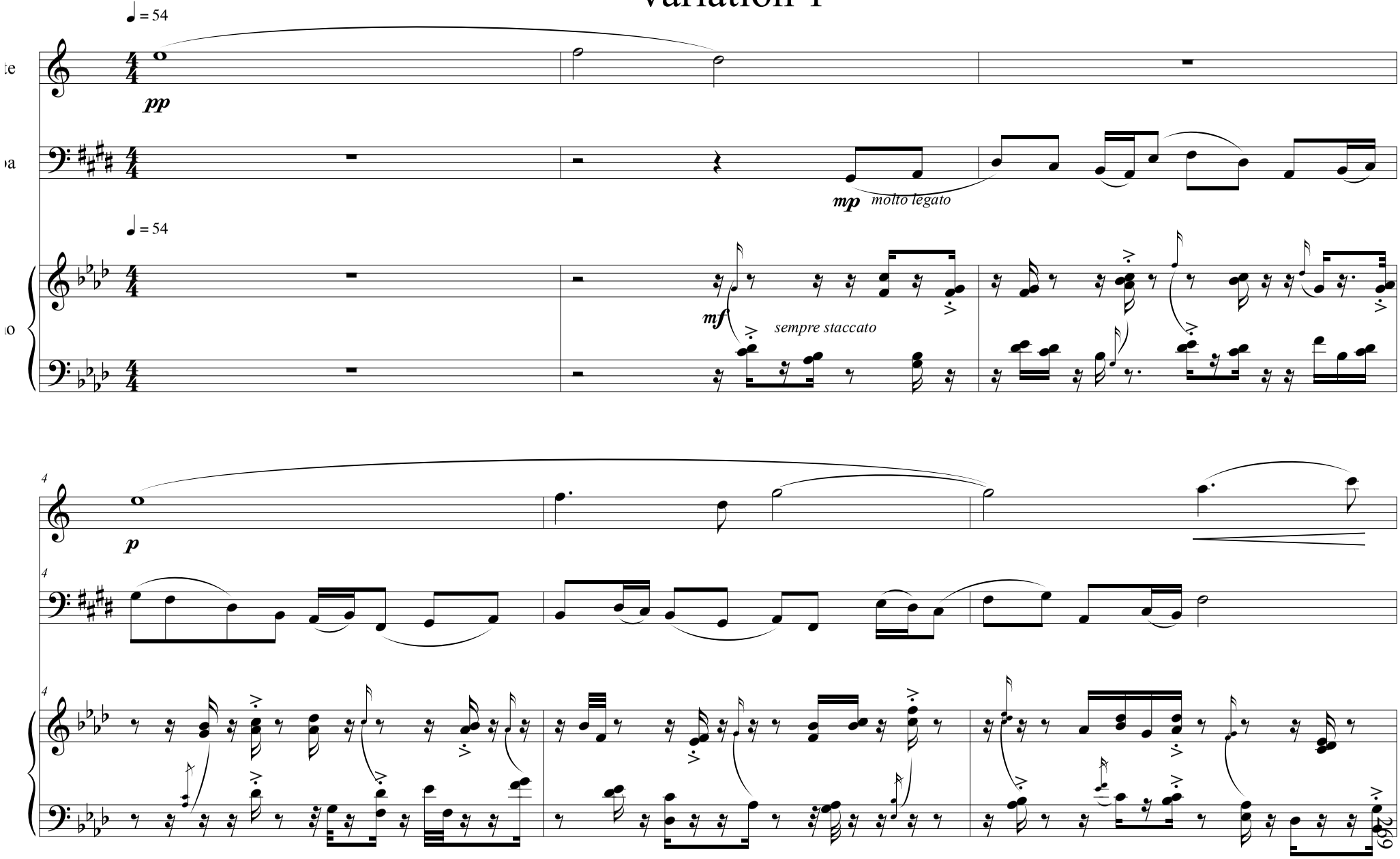
In a performance for educated listeners in the context of Hans Roels's experiment on hyper-polyphony, Variation 3 was additionally performed 'without sharps and flats' (all sharps and flats were ignored by the ers). This resulted in a piece that was perceived as (highly) tonal (or modal) by the listeners. This hat any note combination based on all the pitch classes of a single diatonic 7-set yield a highly sult (see also the experiment with the random 'tonal' generator in Appendix 4).

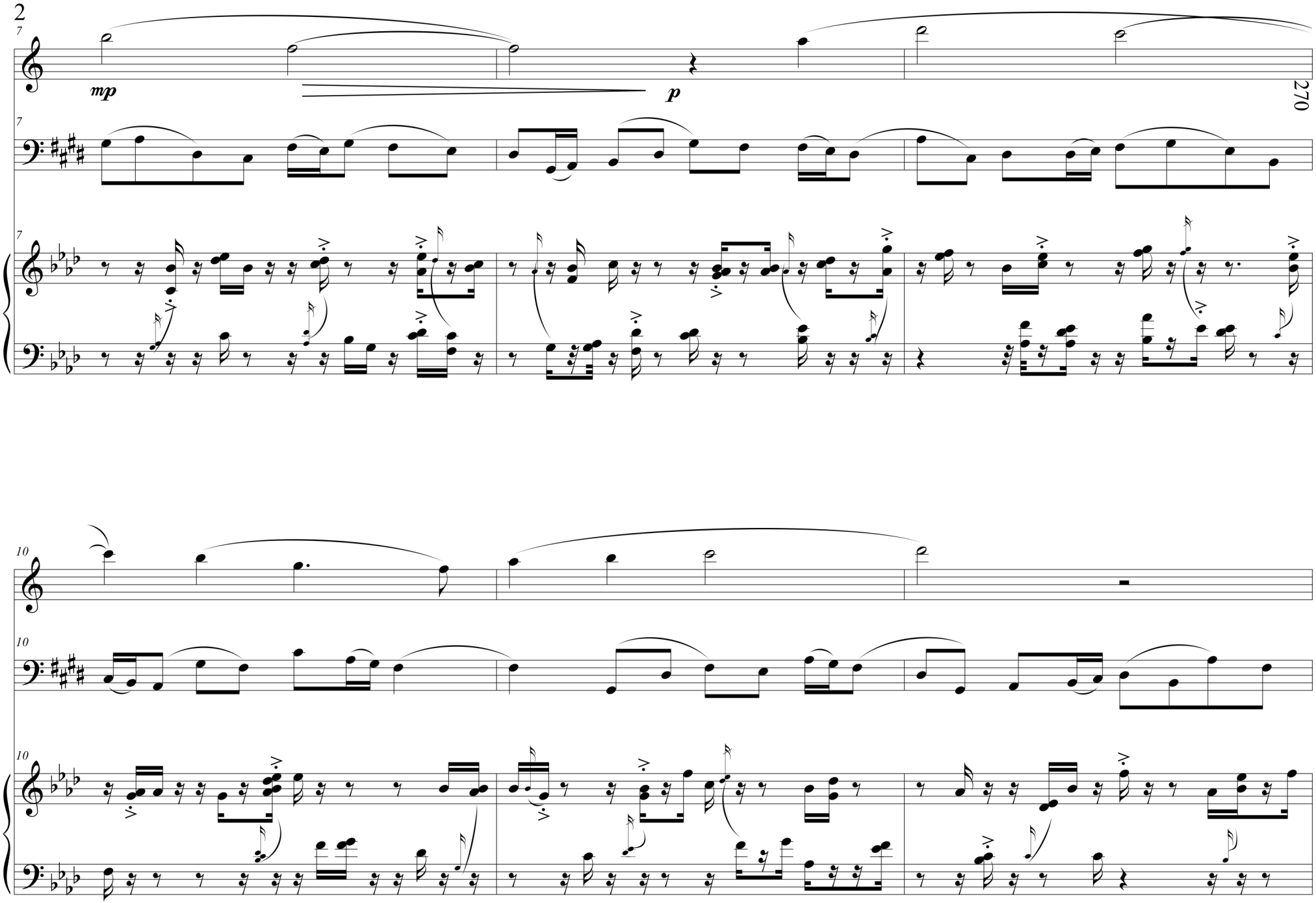
Polytonal variations

3 fragments for flute, bass gamba and piano

Bart Vanhecke

variation 1




7
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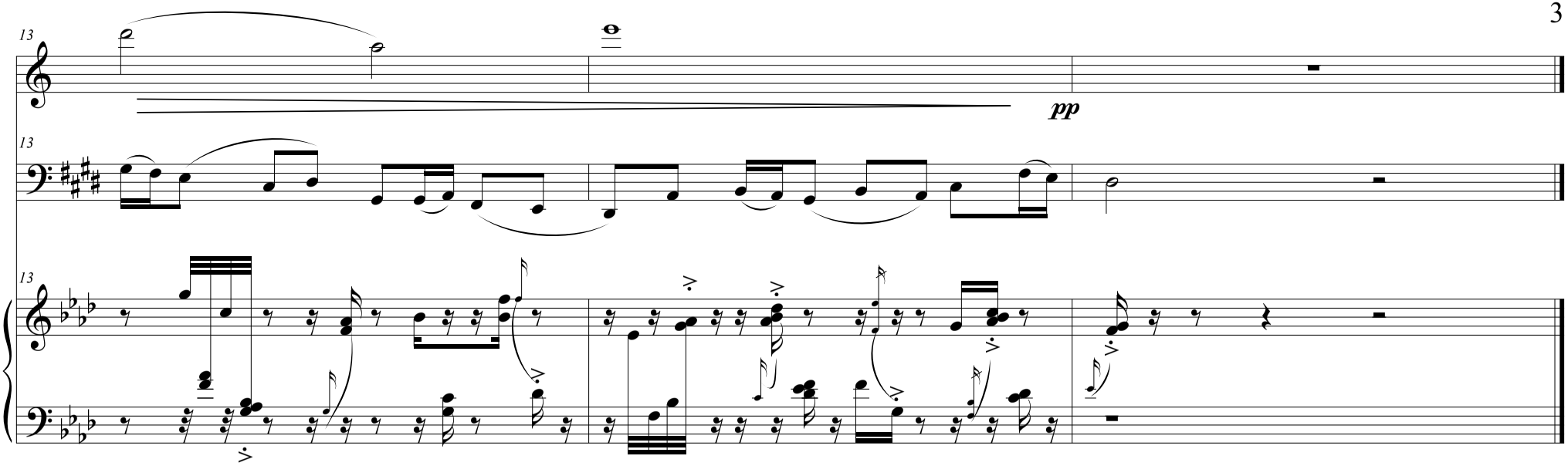
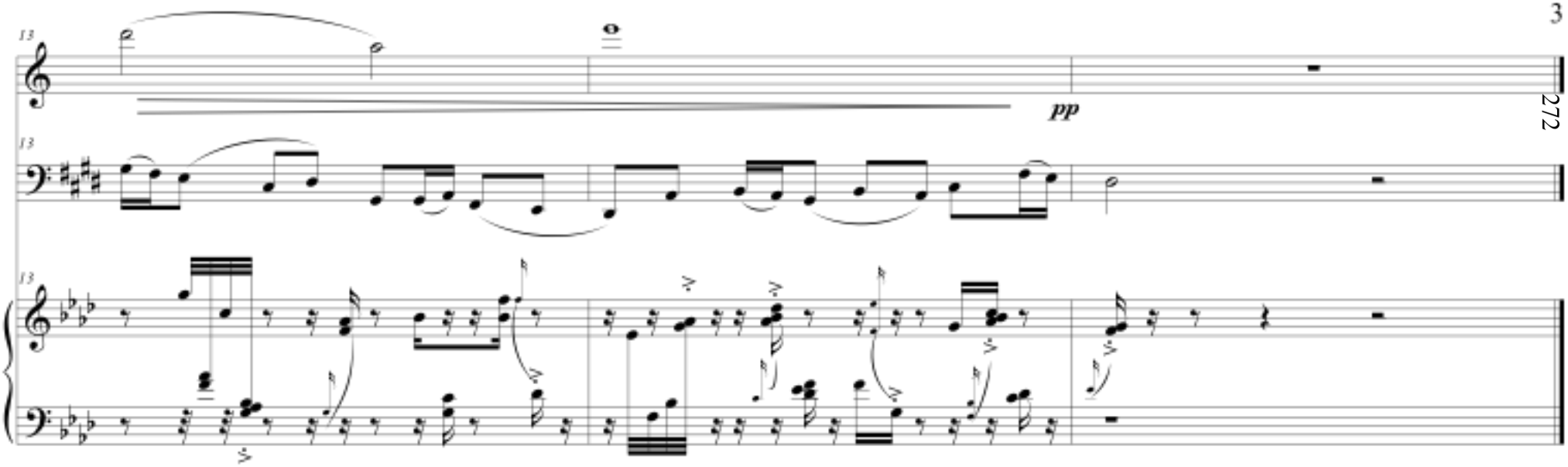
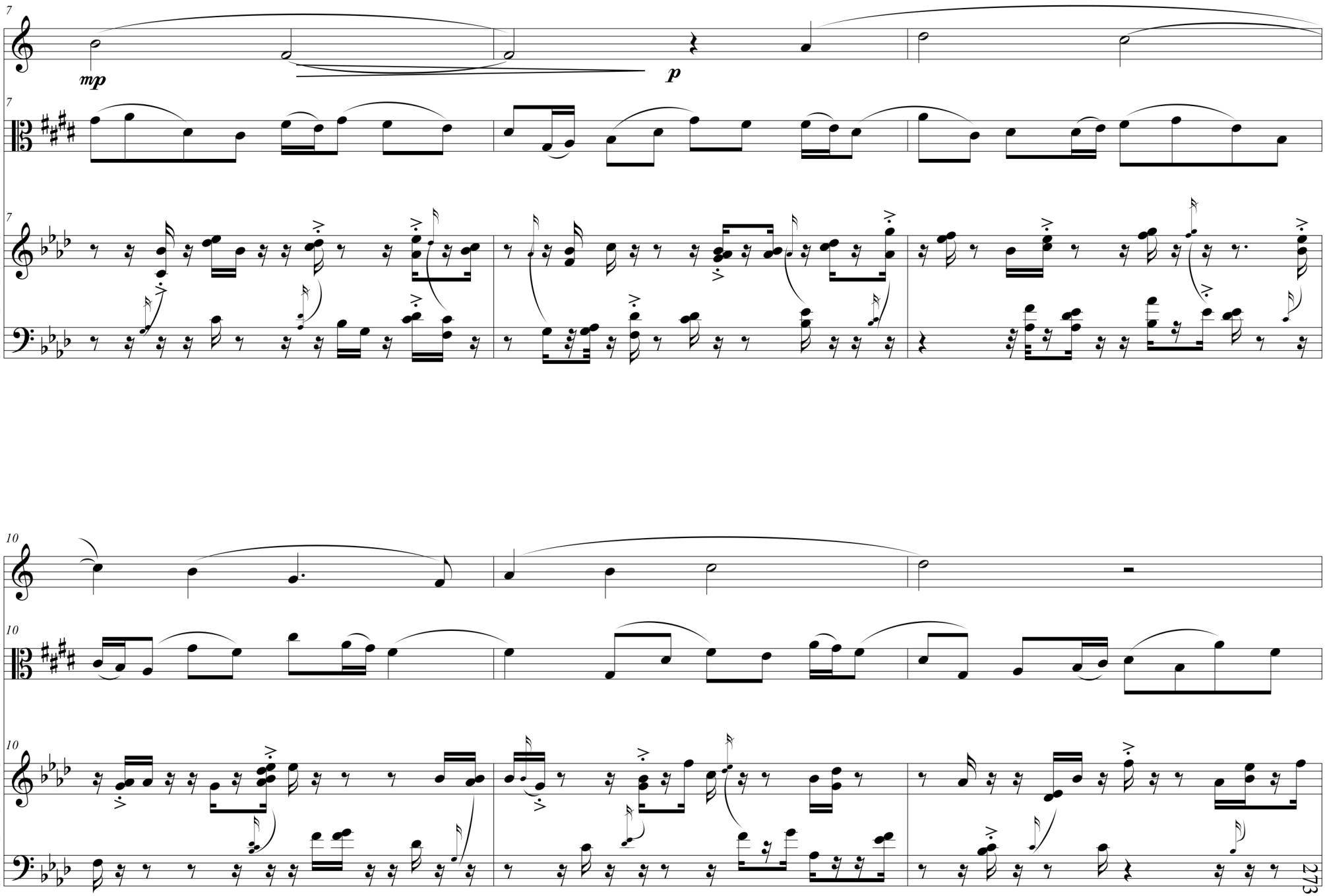
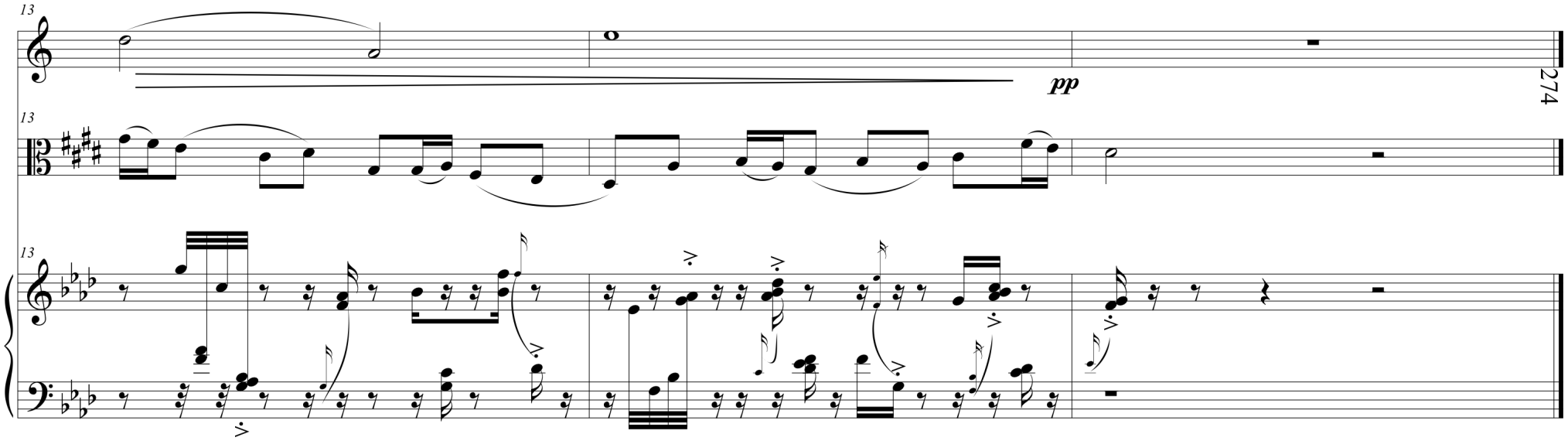
p
270
10

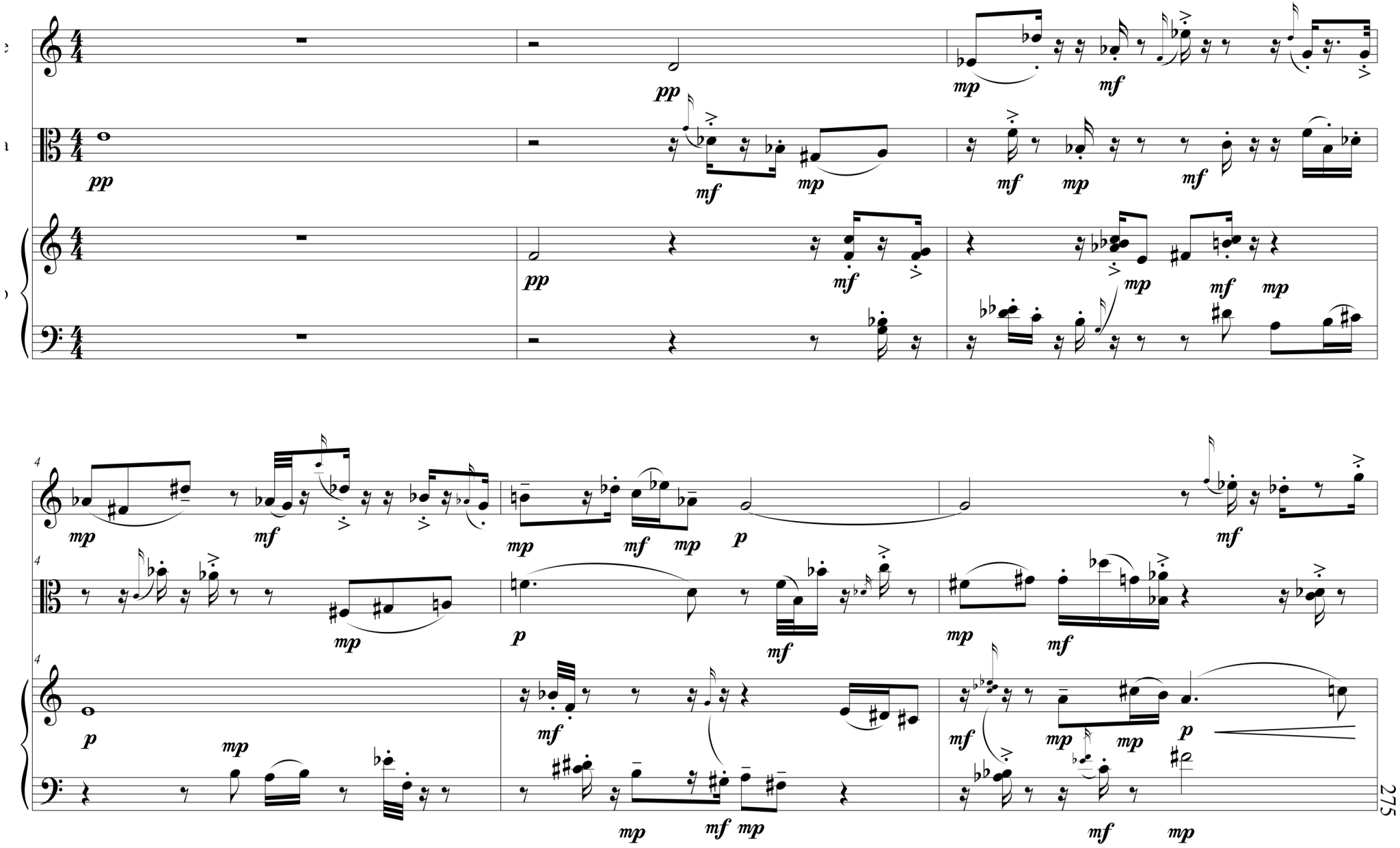

13
pp
13
13


13
pp

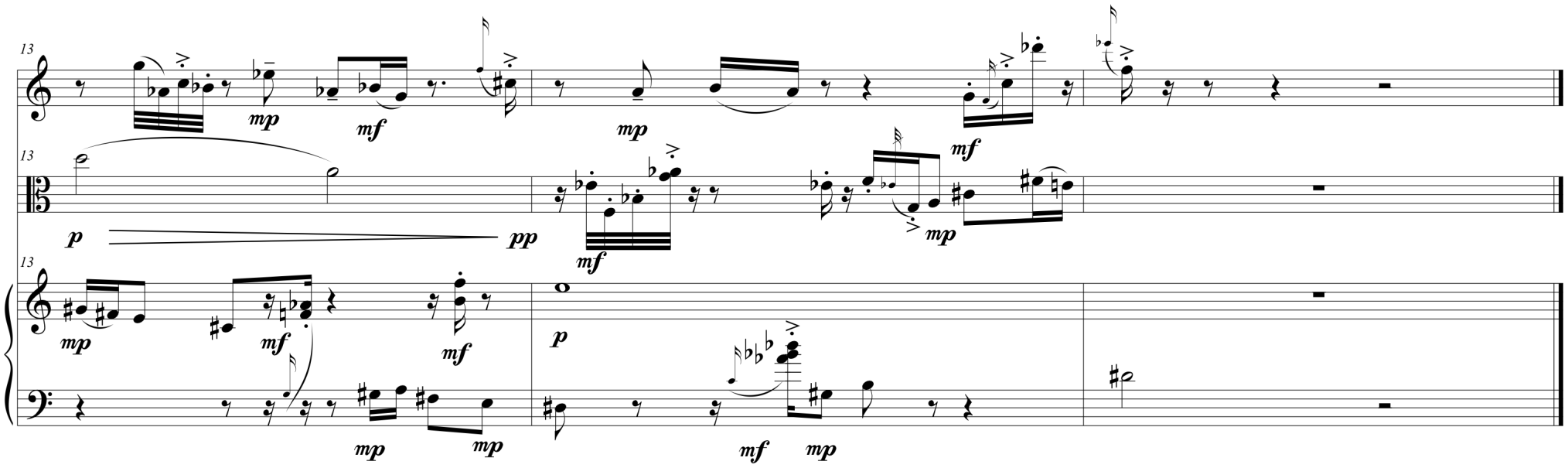

7
mp
p
10


13
pp
274

variation 3

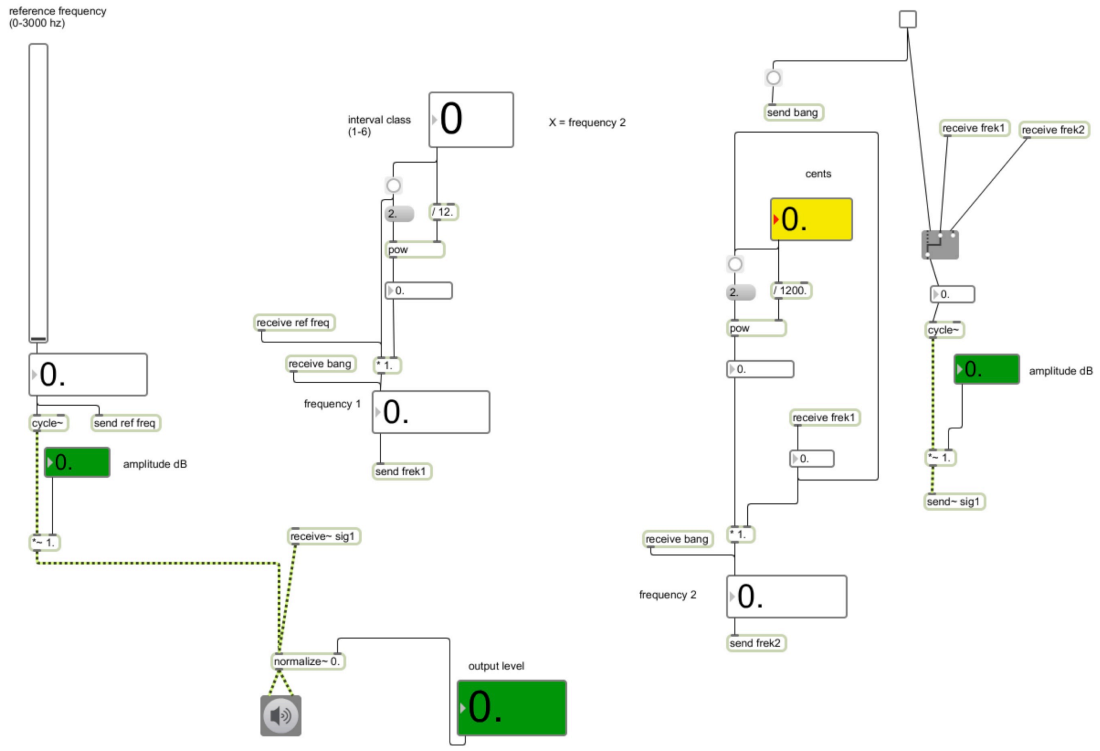


7
mp
mf
fff
p
10


13
mp
mf
mp
mf
13
p
pp
mf
mp
13
mp
mf
mf
p
mp
mp
mf
mp

Appendix 6
Bending-into-place experiment

On 22 February 2011, I performed an experiment at the Orpheus Research Centre in Music (ORCiM) in Ghent (Belgium) to confirm Clarence Barlow's phenomenon of bending into place discussed in Chapter 4. The experiment was performed on the basis of the Max/MSP patch shown below.



Max/MSP patch for the bending experiment.

First, an interval in just intonation was sounded. The participants in the experiment listened to this interval. For instance, a major third (ic 4) of 386,31 cents was heard (see Example 4.11). Then an interval that was further away from the initial interval than the E.T. interval was sounded with the same lowest or highest note as the initial just interval. In the case of ic 4, an interval of 420 cents was sounded. This interval was then gradually reduced or increased towards the just interval. The participants were asked to indicate the moment when they thought the original just interval was reached again. All the participants in the experiment identified the 'just intonation intervals' already before the E.T. interval (of 400 cents in the case of ic 4) was reached. This proves that the 5 cent JND limit is 'careful' and that the participants bend the interval into place.

Appendix 7
List of compositions by Bart Vanhecke

"Camera obscura" (EMS Synthi A. tape. 1989. 15'20'')

"Serenade" (for 2 piccolos, harp and percussion. 1991. ca.6')

"Epitafium" (for alto flute and guitar. 1991. ca.20')

"Monodie" (for piano. 1992 (2° version 1995). ca.10')

"Ombra della sera" (for oboe and piano. 1992. ca.10')

"Twee liederen" (for soprano and 5 instruments (fl., cl., horn, celesta, guit.). 1993. ca.3')

"Quand la lune meurt" (for bass clarinet and ensemble (14 instruments). 1993. ca.20')

"Chaque fleur a une voix" (for bass flute, percussion and live electronics. 1994. ca.22')

"Kwintet" (for woodwinds (alto fl., eng. h., c.b.cl., horn, d. bssn.). 1994. ca.17')

"Tout près de l'eau" (for mezzo-soprano and alto flute. 1995. ca.5')

"La couleur du vent" (for flute. 1996. ca.5')

"Dans les plis des nuages" (for 2 violins and small ensemble (6 instruments). 1996. ca.12')

"Les racines du monde" (for piano. 1998. ca.8')

"Es träumte mir..." (for 6-part male choir. 1998. ca.1'30'')

"Close my willing eyes" (for 3 sopranos and ensemble (9 instruments). 1999. ca.18')

"Etoiles peintes" (string trio. 2000. ca. 9')

"Les fleurs pâles du clair de lune" (for ensemble (12 instruments). 1994 / 2001. ca.10')

"Des cercles sur les eaux" (for harp, ensemble (8 instruments) and live electronics. 2002. ca.15')

"Icarus" (mini opera for six voices and flute. 2004. ca.15')
commissioned by Muziektheatercollectief Walpurgis

"Dans l'eau du songe" (for bass clarinet, cello and piano. 2005. ca.13')
commissioned by Het Collectief

"La hora de la luz" (for countertenor, ensemble (7 instr.) and live electronics. 2005. ca.20')
commissioned by the Spectra Ensemble

"Comme un flocon de neige" (for flute and ensemble (8 instruments). 2007. ca.12')
commissioned by the Ictus Ensemble

"Trinity songs" (for soprano, clarinet and live electronics. 2007. ca.20')
commissioned by Muziektheatercollectief Walpurgis

"Que l'aube apporte la lumière" (piano quintet. complete version 2008 (1° version: 2006). ca.20')
commissioned by the Danel string quartet

"Après la pluie" (for piano and live electronics. 2008. ca. 12'.)

"Le sourire infini des ondes" (for ensemble (9 instruments). 2009.ca14')
commissioned by the Spectra Ensemble

"Danse de la terre" (for orchestra (3,3,3,3/6,3,2,1/5perc,pno/9,9,9,9,9). 2010. ca.12')
commissioned by "Festival van Vlaanderen" (Flanders Festival)

"Un souffle de l'air que respirait le passé" (piano quartet. 2011. ca.15')

"Danse du feu" (for large orchestra (4,4,4,4/4,4,3,1/6 perc,pno,hrp/16,14,12,10,8). 2012. ca.13')

"A l'image du monde… originel" (for piano. 2012. ca.6')

"A l'image du monde… double" (for piano. 2013. ca.6')

"Danse de l'eau et de l'air" (for orchestra (4,3,4,3/4,3,3,1/4perc, pno(&cel),hrp/12,12,10,8,6). 2014. ca.12')

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